

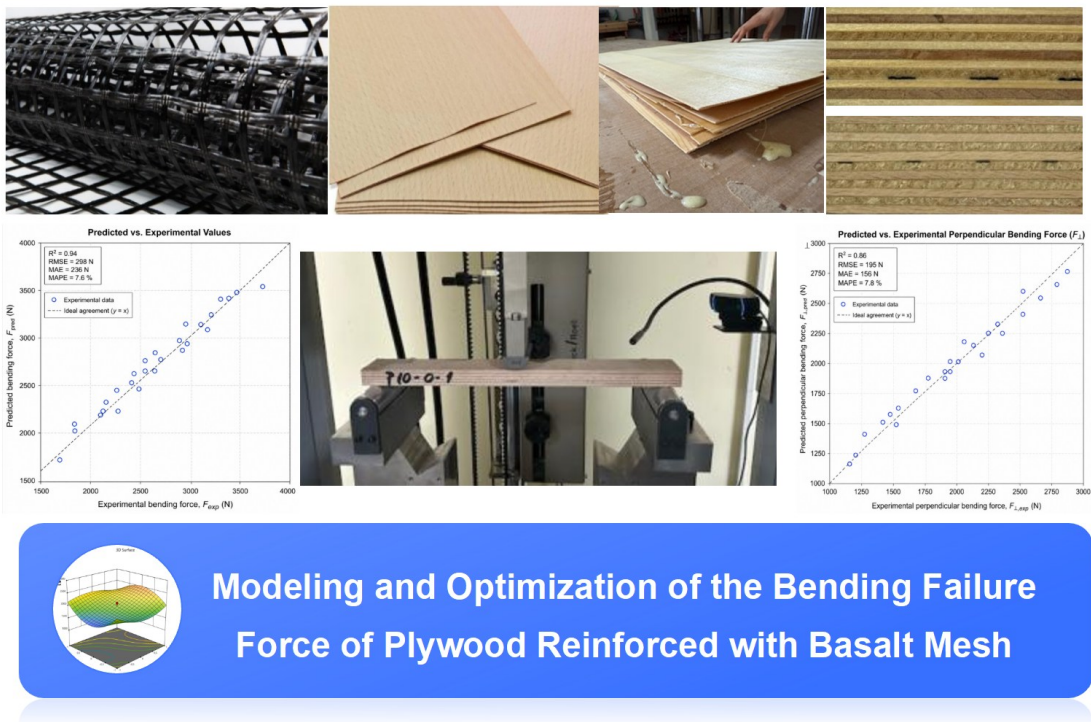
Modeling and Optimization of the Bending Failure Force of Plywood Reinforced with Basalt Mesh

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GRAPHICAL ABSTRACT



Modeling and Optimization of the Bending Failure Force of Plywood Reinforced with Basalt Mesh

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This paper presents the mathematical modeling and optimization of multilayer wood veneer panels (plywood) reinforced with basalt mesh, with the aim of improving their mechanical properties under bending. This study is based on a Central Composite Design (CCD), which analyzed the influence of three factors: panel density, mesh position, and pressing pressure. Based on experimental data, second-order regression models were developed for the maximum bending force parallel to the grain of the surface veneer and perpendicular to the grain of the surface veneer, whose statistical significance was confirmed through analysis of variance (ANOVA). The results indicate a nonlinear influence of the factors and significant interaction effects. It was determined that the bending force parallel to the grain of the surface veneer had an optimum within the experimental space, while the maximum values perpendicular to the grain of the surface veneer occurred at the boundary of the domain. Optimization was carried out using both analytical and numerical methods (MATLAB), with good agreement between the results. Multi-criteria optimization enabled the definition of a compromise solution with balanced mechanical performance.

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Keywords: Wood veneer composites; Basalt mesh; Bending failure force; Mathematical modeling; Optimization

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INTRODUCTION

Plywood has good mechanical properties relative to its density, as well as good thermal and acoustic properties. It is manufactured in different formats, coated with specific films, or surface-treated with agents resistant to biological pests, moisture, and other harmful influences. These properties allow for a wide range of applications, as well as opportunities for further improvement of plywood in order to expand its use even more. One option is reinforcement with different fibers to improve mechanical properties (Čehić and Salah 2018).

A potential application of high-strength plywood is the production of load-bearing floor, wall, and roof elements in timber structures, where low weight, high load-bearing capacity, and good resistance to bending are required. It could be particularly significant for structural elements where reducing the weight of the structure is necessary while simultaneously maintaining or increasing its load-bearing capacity.

The European and American systems do not classify plywood in exactly the same way. The European classification is based on intended use (structural/non-structural),

service conditions, and mechanical properties, whereas in the United States, structural performance categories, intended use, surface/veneer grade, and bond durability are combined. According to EN 636:2012+A1:2015, plywood can be designated according to two basic intended uses: S – structural – for structural or load-bearing applications, and NS – non-structural – for general, non-structural use. For construction products, EN 636 is linked to EN 13986, which further defines the end use of panels, including structural applications in dry, humid, and exterior conditions, as well as non-structural applications such as flooring, roofing, and wall cladding. According to the European standard EN 636, the tested panel can be classified as structural plywood, as it was developed and tested with the aim of achieving increased load-bearing capacity and use in structural elements. The incorporation of basalt mesh further enhances its mechanical properties, particularly its bending load-bearing capacity, thereby confirming its structural character.

The mechanical properties of fiber-reinforced composites mainly depend on the stress transfer mechanism from the matrix to the fibers within the composite. They also depend on fiber length, orientation, proportion, and the direction of the applied load. The use of fiber reinforcements in the veneer industry for broader applications is still relatively limited. For structural materials, common fiber reinforcements include fiberglass, but carbon, aramid, and basalt fibers are also used (Šernek and Jošt 2004; Kufel and Kuciel 2019).

Basalt is used in various forms: fibers, mesh, reinforcement bars, fabrics, pipes, and filaments. In the form of mesh, it is most commonly used in construction for reinforcing various non-wood structures (walls, floor and road substrates, and similar applications). Its low cost and environmental acceptability have contributed to the increasing use of basalt and basalt fibers in the production of certain composite materials used not only in construction, but also in the automotive industry and in the manufacture of wood-based composites.

Basalt fibers use natural basalt as the primary raw material. Their production is based on melting selected basalt rock at a high temperature, after which the molten material is continuously drawn through fine nozzles, forming continuous basalt fibers. The resulting fibers can then be further processed into various forms, such as rovings, yarns, and meshes. Unlike basalt fibers, which are a continuous fibrous product, crushed basalt is a mineral aggregate and cannot directly perform the same reinforcing function in a composite material. The manufacturing process is therefore of particular importance, as it determines the geometry and continuity of the fibers, as well as the final form of the product, and consequently the manner in which the basalt mesh contributes to the mechanical properties of the composite. In this study, a basalt mesh made of continuous basalt fibers, arranged in a mesh structure with regularly defined openings, was used. This enabled the mesh to be positioned between the veneer layers of the plywood and to serve as internal reinforcement.

Wood composites are significant as construction and engineering materials and are widely used in structural and non-structural applications because of their favorable strength-to-weight ratio and sustainability. Recently, increasing attention has been devoted to reinforcing these materials with high-performance fibers, such as basalt fibers, in order to improve their mechanical properties (Daniel and Ishai 2006; Omrani *et al.* 2023). Global studies over the last decade indicate improvements in the mechanical properties of composite panels, including veneer panels reinforced with certain fibers. These studies are focused on fiber content, placement methods within the structure, and the application of environmentally friendly adhesives suitable for bonding of different fibers and wood (Song *et al.* 2017).

When bonding wood to other materials or manufacturing composites with additional reinforcements, epoxy, polyurethane, and other structural adhesives are being increasingly used. These adhesives provide high resistance to moisture, temperature fluctuations, and dynamic loads, making them suitable for structural and load-bearing applications. In contemporary research, particular emphasis is placed on the development of environmentally friendly adhesives with reduced emissions of harmful substances and an improved balance between bond strength and durability. Polyurethane adhesives are used for bonding various materials such as metals, plastics, glass, textiles, and wood. In most cases, they are applied for bonding dissimilar materials, although they can also produce successful bonds between similar materials (Chang *et al.* 2017; Čehić *et al.* 2023). In addition to their many advantageous properties, epoxy adhesives also have certain drawbacks. They are rarely used when only wood bonding is required due to their higher cost and the availability of other adhesive types. However, when bonding wood to other materials, they are often almost indispensable (Yildirim *et al.* 2021).

In their study, (Nunez-Decap *et al.* 2022) investigated the reinforcement of plywood with basalt and carbon engineered fibers, using epoxy resin and polyvinyl acetate as bonding agents to improve its physical and mechanical properties.

In efforts to develop environmentally friendly adhesives, research has focused on the use of alternative reactants instead of formaldehyde. Good results have been achieved using glyoxal, as well as kraft lignin (DKL) and polyethyleneimine (PEI). Optimal results were obtained when the DKL/PEI ratio was 1:1, which improved bond strength (Liu and Li 2006).

Despite numerous studies dealing with the experimental characterization of reinforced wood panels, relatively little attention has been devoted to the development of mathematical models and the optimization of production parameters. In particular, the interaction between structural and technological parameters has not been sufficiently investigated. The aim of this study was to develop a mathematical model and conduct multi-method optimization of plywood reinforced with basalt mesh in order to obtain optimal parameters for the production of this composite material.

MATERIALS AND METHODS

Materials

In this research, multilayer wood veneer panels (plywood) were used as the basic structural material, additionally reinforced with basalt mesh in order to improve their mechanical properties. The panels were made from beech and pine veneers, with the veneer layers mutually oriented at right angles, which is characteristic of plywood structure and enables improved dimensional stability and mechanical resistance.

As reinforcement, a basalt mesh with square openings measuring 10×10 (mm) and fiber thickness of approximately 1 (mm) was used. The basalt fibers from which the mesh is made are obtained by melting natural basalt and are characterized by high tensile strength, chemical resistance, and stability at elevated temperatures. Within the panel structure, the mesh serves the role of absorbing part of the stress, especially in the zones of highest bending loads, thereby contributing to increased load-bearing capacity and fracture resistance.

The mechanical properties of basalt are within the following ranges: Density 2460 to 2960 kg/m³; bending strength 80 to 200 MPa; compressive strength 40 to 20 MPa;

porosity 3.5% to 3.8%; and moisture content 1.1% to 1.8%. Basalts with a density above 2800 kg/m³ contain a certain amount of glassy amorphous phase in their structure (Pavlović *et al.* 2016; Sunny *et al.* 2020).

For bonding the panel layers, polyurethane adhesive Kleiberit PUR 501.0 was used, which ensures adequate adhesion between the wood and the reinforcement. The basic characteristics of this adhesive are: base – isocyanate, color – brown, D4 group, density approximately 1.13 of g/cm³, open time 20 min, consistency – liquid, labeling – labeling required according to EU regulations, contains 4,4-diphenylmethane diisocyanate.

The choice of adhesive is of particular importance due to the need for good stress transfer between the matrix (wood) and the reinforcement (basalt mesh), as well as its influence on the overall mechanical properties of the composite panel. Polyurethane adhesives are used for bonding various materials such as metal, plastic, glass, textiles, and wood (Yildirim *et al.* 2021).

Bonding parameters: Application rate of 200 g/m², open assembly time of 10 to 15 min, pressing pressure 180 to 250 bar, temperature 25 °C, and time pressure 90 min.

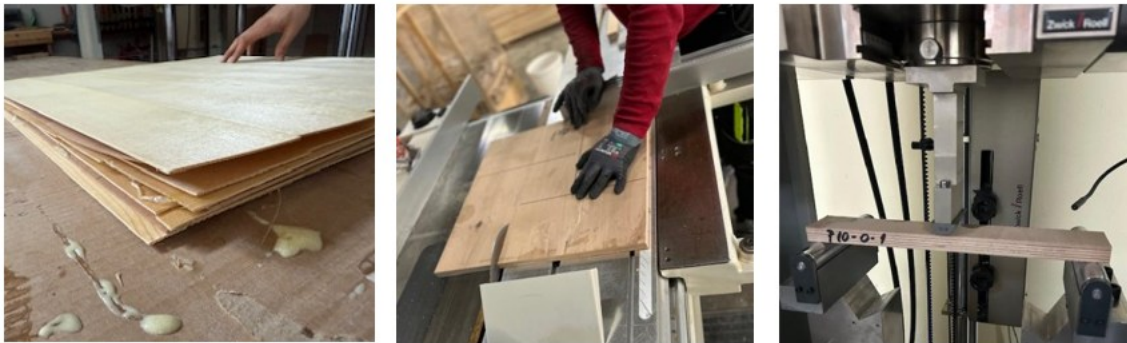
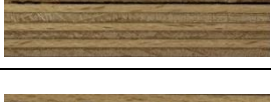
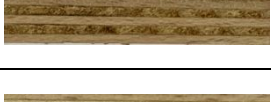
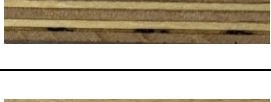
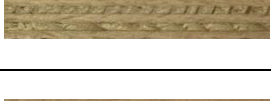
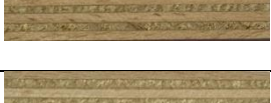
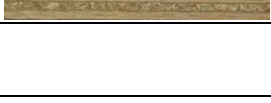


Fig. 1. Production and testing of plywood reinforced with basalt mesh

The three-point bending test for determining the maximum fracture force was conducted in accordance with the EN 310:2001 standard. Table 1 presents the construction of each panel together with the corresponding cross-sectional view.

Table 1. Illustration of the Panel Structure and Layer Arrangement

Panel Type	Visual Representation of the Panel	Panel Structure	Panel Structure Description
P 1		- - - - #- --	The panel consists of eight pine veneer sheets and three beech veneer sheets, with the basalt mesh positioned after the eighth veneer layer.
P 2		- - - - #- --	The panel consists of eight pine veneer sheets and three beech veneer sheets, with the basalt mesh positioned after the eighth veneer layer.
P 3		- - # - - - -	The panel consists of eight pine veneer sheets and three beech veneer sheets, with the basalt mesh positioned after the third veneer layer.

P 4		— —# — — — —	The panel consists of eight pine veneer sheets and three beech veneer sheets, with the basalt mesh positioned after the third veneer layer.
P 5		— — — — —# — —	The panel consists of eight pine veneer sheets and three beech veneer sheets, with the basalt mesh positioned after the eighth veneer layer.
P 6		— — — — —# — —	The panel consists of eight pine veneer sheets and three beech veneer sheets, with the basalt mesh positioned after the eighth veneer layer.
P 7		— —# — — — — —	The panel consists of eight pine veneer sheets and three beech veneer sheets, with the basalt mesh positioned after the third veneer layer.
P 8		— —# — — — — —	The panel consists of eight pine veneer sheets and three beech veneer sheets, with the basalt mesh positioned after the third veneer layer.
P 9		— — — —# — — —	The panel consists of eleven pine veneer sheets, with the basalt mesh positioned after the sixth veneer layer.
P 10		— — — —# — — —	The panel consists of eleven beech veneer sheets, with the basalt mesh positioned after the sixth veneer layer.
P 11		— — — — — — —# —	The panel consists of six beech veneer sheets and five pine veneer sheets, with the basalt mesh positioned after the tenth veneer layer.
P 12		—# — — — — — —	The panel consists of six sheets of beech veneer and five sheets of pine veneer. The basalt mesh is positioned after the first veneer layer.
P 13		— — — —# — — —	The panel consists of six sheets of beech veneer and five sheets of pine veneer. The basalt mesh is positioned after the sixth veneer layer.
P 14		— — — —# — — —	The panel consists of six sheets of beech veneer and five sheets of pine veneer. The basalt mesh is positioned after the sixth veneer layer.
P 15		— — — —# — — —	The panel consists of six sheets of beech veneer and five sheets of pine veneer. The basalt mesh

			is positioned after the sixth veneer layer.
P 16		— — — — — — —	The panel consists of six sheets of beech veneer and five sheets of pine veneer. The basalt mesh is positioned after the sixth veneer layer.
P 17		— — — — — — —	The panel consists of six sheets of beech veneer and five sheets of pine veneer. The basalt mesh is positioned after the sixth veneer layer.
P 18		— — — — — — —	The panel consists of six sheets of beech veneer and five sheets of pine veneer. The basalt mesh is positioned after the sixth veneer layer.
P 19		— — — — — — —	The panel consists of six sheets of beech veneer and five sheets of pine veneer. The basalt mesh is positioned after the sixth veneer layer.
P 20		— — — — — — —	The panel consists of six sheets of beech veneer and five sheets of pine veneer. The basalt mesh is positioned after the sixth veneer layer.
Note: —/ - Color blue sheet for pine veneer; —/ -color red sheet of beech veneer; and # - basalt mesh			

Central Composite Design for Experiments

In order to analyze the influence of structural and technological parameters on the mechanical properties of the tested panels, a Central Composite Design (CCD) was applied. This approach enables efficient modeling of nonlinear relationships between input factors and output responses with a relatively small number of experiments. Considering the objective of the research and the realistic technical and technological conditions of plywood production, the following independent variables were selected in this study:

X_1 - panel density ρ_p , depending on the type of wood used,

X_2 - position of the mesh within the structure m ,

X_3 - specific pressing pressure p .

The ranges of factor variation were defined based on preliminary research results, thereby determining the realistic boundaries of the experimental space (Talić and Čehić 2024; Talić *et al.* 2025). The factors were coded into a dimensionless interval according to the standard transformation,

$$X_i = \frac{x_i - x_{0i}}{\Delta x}, i = 1, 2, 3 \quad (1)$$

where X_i is the coded value of the factor, x_i is the real value, x_0 is the central value, and Δx is the half-range of factor variation (Krstić 2006; Jurković 1999).

For the purposes of this experiment, a rotatable design was used, which is a special form of the CCD method and is therefore frequently applied in modeling and adaptive control of multivariable processes. This design contains the basic factorial part of the

design, symmetrically positioned axial points around the design center, and repeated center points. In general form, the number of experiments is obtained from Eq. 2 (Jurković 1999),

$$N = 2^{k-p} + 2k + n_0 = n_k + n_\alpha + n_0 \quad (2)$$

where N is the total number of experiments, n_k is the number of variable factors, n_0 is the number of repetitions at the center point, and n_α is the number of symmetrically positioned axial points around the center point.

In this specific case, $k = 3$ (variable factors: ρ , m , p), so the total number of experiments is $N = 20$. Based on this, the number of factorial design points was determined: $n_k = 2^k = 2^3 = 8$. This gives points corresponding to all combinations of minimum and maximum values for the three factors.

The number of axial points was determined: $n_\alpha = 2 \cdot 3 = 6$. Axial points are positioned along each axis at a distance of $\pm\alpha$ from the center, where α is the value ensuring rotatability. For a rotatable design, α is calculated as: $\sqrt[4]{2^k} = \sqrt[4]{8} \approx 1.682$.

The number of center points was determined: The number of center points n_0 may be any number, but it is usually set as one or more points to enable estimation of experimental error. In this case, for rotatability, $n_0 = 6$ was selected.

The experimental design included a total of 20 experiments, which were obtained as follows:

$$N = 2^{k-p} + 2k + n_0 = n_k + n_\alpha + n_0 = 8 + 6 + 6 = 20 \quad (3)$$

The rotatability of the design ensures that the prediction variance depends solely on the distance from the center point and not on the direction in factor space, thereby achieving uniform model precision throughout the entire experimental domain. Based on the defined experimental design, an experiment matrix was formed and used for conducting tests and collecting data for further regression analysis and process optimization. The selected CCD design enables the development of a second-order quadratic model suitable for describing complex nonlinear systems and identifying optimal operating conditions.

Mathematical Modeling

In order to quantitatively describe the influence of the selected factors on the mechanical characteristics of the tested panels, mathematical models were developed using Response Surface Methodology (RSM). This approach enables approximation of the real relationship between input variables and output responses by means of second-order polynomial functions.

It was necessary to define an adequate and reliable mathematical model of the dependence of the maximum fracture force $F_{\max, \perp, \text{II}}$ on panel density ρ_p (kg/m³), mesh position m (layer number), and pressing pressure p (bar). If the input and output physical quantities are replaced by independent and dependent mathematical variables, and the function $F_{\max} = f(\rho, m, p)$ is expressed in polynomial form, then the formalized description of the dependence of the fracture force $F_{\max, \perp, \text{II}}$ on panel density ρ_p , mesh position m , and pressing pressure takes the mathematical form $Y = f(X_1, X_2, X_3)$. By replacing input-output physical quantities with independent-dependent variables and expressing the process state function $F = f(\rho, p, m)$ in polynomial form, the formalized description becomes:

$$Y = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_{11}X_1^2 + b_{22}X_2^2 + b_{33}X_3^2 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{23}X_2X_3 + b_{123}X_1X_2X_3 \quad (4)$$

where: $Y = F$ - dependent variable corresponding to the physical, *i.e.*, experimental value of bending fracture force, X_1, X_2, X_3 - independent variables corresponding to physical values (ρ, m, p), $b_0, b_1, b_2, b_3, b_{11}, b_{22}, b_{33}, b_{12}, b_{13}, b_{23}, b_{123}$ - regression coefficients of the mathematical model (Jurković 1999; Jurković *et al.* 2011).

The regression coefficients were determined using the least squares method, based on experimental results obtained according to the defined CCD design. Before modeling, the response variables were transformed using a logarithmic function to achieve better linearization of the relationships and model stability: $Y = \ln(F)$. This reduced variance heterogeneity and improved the accuracy of regression parameter estimation. Based on experimental data, two separate models were developed to describe the maximum bending force: parallel to the grain of the surface veneer ($F_{\max \parallel}$), perpendicular to the grain of the surface veneer ($F_{\max \perp}$).

The homogeneity test of experimental results was carried out in order to determine whether the variances of repeated measurements were equal, *i.e.*, whether the experimental error could be considered constant throughout the entire testing domain. This verification is an important prerequisite for applying parametric statistical methods such as regression analysis, Fisher's criterion, and Student's t-test.

Since for $F_{\max \parallel}$ ($Kh = 0.385 < Kt = 0.455$) and $F_{\max \perp}$ ($Kh = 0.340 < Kt = 0.455$), the experiment could be continued because the dispersion was homogeneous. Further calculations yielded the mathematical models for the two forces ($F_{\max \parallel}$ and $F_{\max \perp}$), which have the following form:

$$Y_{\parallel} = 8.0385 + 0.206 X_1 + 0.0145 X_2 + 0.0009 X_3 - 0.138 X_1^2 - 0.0635 X_2^2 - 0.0455 X_3^2 + 0.0228 X_1 X_2 + 0.0126 X_1 X_3 + 0.005 X_2 X_3 - 0.0148 X_1 X_2 X_3 \quad (4)$$

$$Y_{\perp} = 7.587 + 0.0915 X_1 + 0.0580 X_2 + 0.0183 X_3 - 0.0368 X_1^2 - 0.01434 X_2^2 - 0.0592 X_3^2 + 0.0039 X_1 X_2 + 0.0283 X_1 X_3 - 0.0262 X_2 X_3 - 0.06166 X_1 X_2 X_3 \quad (5)$$

If only significant parameters are considered and parameter decoding is performed, the following equations are obtained:

$$Y_{\parallel} = \exp(Y_{\parallel}) = 3.72148 + 0.015649p + 0.018571m + 6.553209\rho + 0.0279602pm - 4.211011\rho^2 - 0.0031295m^2 - 0.000035p^2 \quad (6)$$

$$Y_{\perp} = \exp(Y_{\perp}) = 6.60929 + 0.0127979p - 0.0000464p^2 - 0.2940814m + 0.001399mp - 2.004984\rho + 0.011444\rho p + 0.464429\rho m - 0.0021172\rho mp \quad (7)$$

The multiple regression coefficient for $F_{\max \parallel}$ is $R = 0.83$, while for $F_{\max \perp}$ it is $R = 0.90$, indicating how strong the overall relationship is between experimentally measured values and values predicted by the regression model. In this case, the relationship is considered strong.

The experimental results and the predictions of the mathematical model are presented graphically, showing the relationship between the experimentally measured and model-predicted values of the maximum bending force parallel ($F_{\max \parallel}$) and perpendicular ($F_{\max \perp}$) to the grain. In both cases, a good agreement between the experimental and predicted values can be observed, confirming the adequacy and predictive capability of the developed second-order regression models.

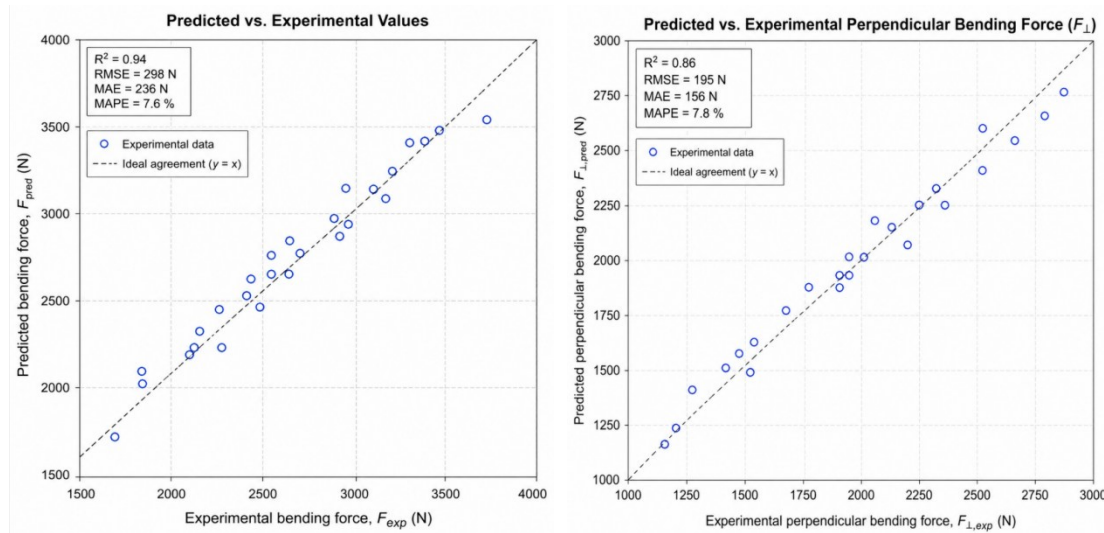


Fig. 2. Comparison of experimental and predicted bending force values

The models obtained enable analysis of the individual influence of factors, their mutual interactions, as well as identification of nonlinear effects within the experimental space. The quadratic terms are of particular importance because they indicate the existence of extreme values (minimum or maximum), which is crucial for process optimization.

Optimization Methods

In order to determine the optimal values of the input parameters, two complementary optimization methods were applied: analytical optimization based on differential calculus, and numerical optimization using the MATLAB software environment. This approach enables verification of result consistency and increases the reliability of the obtained optimal solutions.

Analytical Optimization

Analytical optimization was based on determining the stationary points of the developed second-order regression models. For this purpose, the partial derivatives of the response function with respect to all independent variables were set equal to zero:

$$\frac{\partial F}{\partial x_j} = 0; \quad (j = 1, 2, 3, \dots, n) \quad (8)$$

By solving the resulting system of equations, the coordinate values of the stationary point in the coded factor space were determined. After that, the stationary point was classified based on the second-order Hessian matrix, which established whether the point represented a maximum, minimum, or saddle point. The obtained optimal factor values in coded form were transformed into real values using the inverse transformation:

$$x_i = X_i \cdot \Delta x + x_0 \quad (9)$$

The analytical method enables a clear insight into the model structure and direct determination of extreme values, but it is limited to functions that are differentiable and relatively simple to solve (Myers *et al.* 2016).

Numerical Optimization (MATLAB)

In addition to the analytical approach, optimization was also performed using numerical methods in the MATLAB software environment. For this purpose, the `fmincon` function was applied, enabling the solution of nonlinear optimization problems with defined constraints.

The optimization problem was formulated as the minimization of the negative value of the response function: $\min(-Y)$, thereby transforming the maximization problem into a standard form suitable for numerical solving. The variable boundaries were defined in accordance with the experimental design: $-1.682 \leq x_i \leq 1.682$.

Numerical optimization begins from an initial point within the permitted domain and iteratively searches for the optimal solution using gradient methods. The optimization process is characterized by gradual reduction of the objective function value and convergence toward a local optimum, while satisfying all imposed constraints.

The advantage of the numerical approach lies in the possibility of solving more complex problems and the flexibility in defining constraints, while its disadvantage may be sensitivity to the choice of the initial point and the possibility of finding a local optimum (Myers *et al.* 2016).

The combination of analytical and numerical optimization enables reliable verification of the obtained results and contributes to greater confidence in defining optimal production parameters.

RESULTS AND DISCUSSION

Based on the experimental results and application of the CCD method, second-order regression models were developed to describe the maximum bending force parallel to the grain of the surface veneer ($F_{\max \parallel}$) and perpendicular to the grain of the surface veneer ($F_{\max \perp}$). The models were formed in the coded factor space using Response Surface Methodology.

Statistical analysis showed that the developed models achieved satisfactory accuracy and predictive capability within the defined experimental space. Analysis of variance (ANOVA) confirmed the statistical significance of the models, where certain linear, quadratic, and interaction terms showed dominant influence on the analyzed responses.

The obtained results indicated pronounced nonlinear system behavior, confirming the justification for applying quadratic regression models. Particularly significant were the quadratic terms of the models, which indicated the existence of extreme response values within the experimental domain.

For the bending force parallel to the grain of the surface veneer ($F_{\max \parallel}$) the existence of an internal optimum was observed, indicating that the maximum response value was not tied to the boundaries of the experimental space. On the other hand, for the bending force perpendicular to the grain of the surface veneer ($F_{\max \perp}$) the maximum values occurred near the boundaries of the defined factor domain, indicating a different nature of the response and greater system sensitivity to changes in input parameters.

Results of Analytical Optimization

Analytical optimization was performed by determining the stationary points of the developed regression models. The partial derivatives of the response functions with respect

to all independent variables were set equal to zero, thereby forming a system of equations for determining the optimal factor values.

Analysis of the Hessian matrix showed that the model for the bending force parallel to the grain of the surface veneer ($F_{\max \text{ II}}$) possessed a maximum within the experimental space. The obtained optimal values indicated that a combination of medium and higher levels of the analyzed factors enabled achievement of the highest bending load-bearing capacity in this direction. Accordingly, the maximum bending fracture force for $F_{\max \text{ II}}$ was obtained for the following parameter values:

x_1 - panel density $\rho = 0.791 \text{ (g/cm}^3\text{)}$,

x_2 - mesh position $m = 4.87 \text{ (layer)}$,

x_3 - pressing pressure $p = 215 \text{ (bar)}$.

For the bending force perpendicular to the grain of the surface veneer ($F_{\max \perp}$) analytical analysis showed that the optimum was not completely enclosed within the experimental space, but rather that the maximum values approached the boundary values of certain factors. Such behavior indicates the possibility of further increasing the response by expanding the experimental domain. Since there were one positive and two negative values, the Hessian was indefinite, meaning the stationary point was a saddle point rather than a maximum or minimum. Therefore, the maximum was sought at the boundary (within the factor constraints). Accordingly, the maximum bending fracture force for $F_{\max \perp}$ was obtained for the following parameter values:

x_1 - panel density $\rho = 0.820 \text{ (g/cm}^3\text{)}$,

x_2 - mesh position $m = 6.389 \text{ (layer)}$,

x_3 - pressing pressure $p = 195 \text{ (bar)}$.

The analytical approach enabled a clear insight into the mathematical structure of the model and the mutual influence of individual factors, which represents a significant advantage in result interpretation.

Numerical Optimization Using MATLAB

Numerical optimization was carried out using the `fmincon` function in the MATLAB environment. The optimization problem was formulated as the minimization of the negative value of the response function, with constraints defined by the experimental design. The optimization results showed good algorithm convergence and stability of the numerical procedure. The iterative optimization process led to optimal parameter values that were in good agreement with the results obtained by the analytical method.

The following presents the analysis of individual optima for the force parallel to the grain of the surface veneer ($F_{\max \text{ II}}$), the force perpendicular to the grain of the surface veneer ($F_{\max \perp}$), as well as the joint optimum obtained using the desirability method:

1. Optimum only for $F_{\max \text{ II}}$

Optimal parameter values: $x_1 = 0.7909 \text{ (g/cm}^3\text{)}$, $x_2 = 4.87 \text{ (layer)}$, $x_3 = 215 \text{ (bar)}$.

Predicted maximum force value: $F_{\max \text{ II}} = 3358.83 \text{ N}$.

This optimum represents a realistic and stable solution within the CCD space, without pronounced model extrapolation.

2. Optimum only for $F_{\max \perp}$

Optimal parameter values: $x_1 = 0.905$ (g/cm³), $x_2 = 1.29$ (layer), $x_3 = 181.36$ (bar).

Predicted maximum force value: $F_{\max \perp} = 40340.83$ N.

This value represents the mathematical maximum of the model at the boundary of the CCD space. Due to the logarithmic transformation and the strong triple interaction term, the obtained value represents extrapolation and is not physically realistic. Therefore, the joint optimum is used.

3. Joint Optimum (Desirability)

Optimal parameter values: $x_1 = 0.817$ (g/cm³), $x_2 = 3.69$ (layer), $x_3 = 203.7$ (bar).

Predicted maximum force value: $F_{\max \parallel} = 3251.68$ N, $F_{\max \perp} = 2822.50$ N.

Desirability = 0.917 (very high simultaneous satisfaction of both objectives)

This solution represents a compromise optimal state that enables high values of both forces while maintaining model stability and realism. The advantage of MATLAB optimization lies in the possibility of efficiently solving complex nonlinear problems, as well as in the simple implementation of constraints and numerical procedures.

Comparison of Optimization Methods

By comparing the results obtained through analytical and numerical optimization, good agreement between the applied methods was observed. The differences between the optimal values were relatively small and mainly resulted from the different mathematical approaches used for determining the optimum. The analytical method enabled detailed understanding of model behavior and identification of stationary points, while numerical optimization provided greater flexibility in solving problems with constraints and more complex nonlinear relationships. The combination of these two approaches enabled additional verification of the obtained results and increased the reliability of the defined optimal process parameters.

The following table presents the optimization results obtained using both methods: classical mathematical analysis and numerical optimization using MATLAB.

Table 2. Optimization Results by Methods

Optimization Methods	Panel Density - X_1 ρ_p (g/cm ³)	Mesh Position - X_2 m (layer)	Pressing Pressure - X_3 p (bar)	$F_{\max \parallel}$ R_1 (N)	$F_{\max \perp}$ R_2 (N)	Note
Classical mathematical analysis only $F_{\max \parallel}$	≈ 0.79	≈ 4.87	215	≈ 3358	—	Stationary point ($\nabla F=0$)
Classical mathematical analysis only $F_{\max \perp}$	≈ 0.82	≈ 6.389	≈ 195	—	Maximum at the boundary	Boundary optimum
MATLAB – only $F_{\max \parallel}$	0.7909	4.87	215	3358.83	—	fmincon
MATLAB – only $F_{\max \perp}$	0.9052	1.29	181.36	—	40340.83*	Axial point (± 1.682)
MATLAB – joint optimum	0.8172	3.69	203.71	3251.68	2822.50	D = 0.917

Classical mathematical analysis and MATLAB (fmincon) optimization for $F_{\max II}$ practically coincided for X_1 , X_2 , X_3 , which was also confirmed by the obtained $F_{\max II}$ (3358 N and 3358.83 N, respectively). This indicates that the model was consistent and that the optimum was obtained as a stationary point ($\nabla F = 0$).

For $F_{\max \perp}$ the optimum appeared as an “extreme on the boundary” (edge of the design/axial point), which explains why this method did not produce an “internal” stationary point, but rather the maximum “moved” toward the boundary of the search space (here $X = \pm 1.682$).

The joint optimum (MATLAB) with $D = 0.917$ represented the best compromise: it slightly reduced $F_{\max II}$ compared to its maximum value, while simultaneously increasing $F_{\max \perp}$ compared to other compromise settings.

When only $F_{\max II}$ was optimized, a high result was obtained (3358.83), but without controlling the behavior of $F_{\max \perp}$. When only $F_{\max \perp}$ was optimized, the optimum appears at the axial boundaries of the experimental space, indicating that the model continued to increase in that direction and that the optimum lay at the edge of the design. This means that individual optimization produced extreme values, but it was not suitable for realistic engineering applications where both responses are important.

Individual optimization led to extreme solutions. Joint optimization provided a stable and engineeringly acceptable compromise. If the goal is to simultaneously optimize both forces and obtain one “best” solution for the process, then the most logical choice is the joint optimum (MATLAB, Desirability):

- Parameters: $x_1 = 0.817$ (g/cm³), $x_2 = 3.69$ (layer), $x_3 = 203.71$ (bar).
- Predicted maximum force values: $F_{\max II} = 3251.68$ N, $F_{\max \perp} = 2822.50$ N.
- Desirability: $D = 0.917$.

Selecting this solution provided the best balance between increasing the force in the direction of the surface veneer grain and perpendicular to the surface veneer grain, without “forcing” the parameters to the design boundaries. The obtained results confirm that the position of the basalt mesh, panel density, and pressing pressure had a significant influence on the mechanical behavior of multilayer wood panels. The influence of mesh position is particularly pronounced, since proper positioning of the reinforcement enables more efficient absorption of bending stresses and increases the load-bearing capacity of the structure.

Increasing panel density generally contributes to higher mechanical resistance, but it may also affect the internal stress distribution and system stiffness. Likewise, pressing pressure showed a significant influence on the quality of the bonded joint and the overall homogeneity of the composite structure.

The research results demonstrate that mathematical modeling and optimization represent an efficient tool for defining optimal production parameters and improving the performance of wood composite materials reinforced with basalt mesh.

CONCLUSION

This study presented the mathematical modeling and optimization of multilayer wood veneer panels (plywood) reinforced with basalt mesh, with the aim of analyzing the influence of structural and technological parameters on bending mechanical characteristics.

1. By applying the Central Composite Design and Response Surface Methodology, second-order regression models were developed to describe the maximum bending force parallel and perpendicular to the grain of the surface veneer.
2. The developed mathematical models showed satisfactory accuracy and predictive capability within the defined experimental space. Statistical analysis confirmed the significance of the models and justified the application of quadratic regression functions for describing the analyzed system.
3. It was determined that the analyzed factors — panel density, basalt mesh position, and pressing pressure — had a pronounced influence on the mechanical characteristics, while significant interaction effects between individual parameters were also identified.
4. For the bending force parallel to the grain of the surface veneer, the existence of an internal optimum within the experimental space was established, indicating the possibility of defining an optimal combination of structural and technological parameters. For the bending force perpendicular to the grain of the surface veneer, the maximum response values occurred near the boundaries of the defined factor domain, indicating increased system sensitivity and the possibility of further improving mechanical properties by expanding the experimental range.
5. Analytical optimization enabled the determination of stationary points and detailed understanding of the mathematical structure of the model, while numerical optimization in the MATLAB environment demonstrated high efficiency in solving nonlinear optimization problems. By comparing the results obtained using different optimization methods, good agreement between the analytical and numerical approaches was confirmed, further verifying the reliability of the defined optimal solutions.
6. The optimization was restricted to the experimental design domain, and predicted responses outside this domain were not considered reliable without additional experimental validation.

The results obtained confirm that the application of basalt mesh can significantly contribute to improving the mechanical performance of multilayer wood panels, especially under bending loads. The developed models and defined optimal parameters may serve as a basis for further development and application of basalt mesh-reinforced wood panels in modern structural systems.

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