Numerical Calculation Method of Dynamic Responses for Wood Structures with Frequency-related Damping Parameter

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The loss factor of wood material is frequency related, which directly affects the calculation method of dynamic responses for wood structures. In this paper, the relationship between loss factor and damping coefficient was determined based on equal dissipated energy. Combined with the time-domain and frequency-domain methods, a modal superposition method was proposed to calculate the dynamic response of wood structures. Compared with the frequency-domain method, the proposed method can additionally consider the transient vibration responses of wood structures. Compared with the equivalent time-domain method based on constant loss factor, the proposed method can additionally consider the influence of frequency related loss factor. The proposed method should be preferred to calculate dynamic responses of wood structures.

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INTRODUCTION

Wood, as a natural building material, has the advantages of being environmentally friendly and easy to process (Nam *et al.* 2023). Wood structures have good energy dissipation performance and play an effective role in earthquake resistance and wind resistance (Sun *et al.* 2024a; Vutukuru *et al.* 2024; Jensen *et al.* 2025). However, the damping parameter of wood material is frequency related (Zhang and Zhou 2023). Ouis (2003) analyzed the loss factor of wood with different species, and found a dependence on vibration frequency. Elie *et al*. (2013) analyzed the loss factors in the low- and the mid-frequency domains based on the subspace method, respectively. The loss factor of wood structures has been found to vary with frequency. It is important to know how to calculate the dynamic responses of wood structures. The traditional time-domain calculation methods cannot be directly used to calculate the dynamic responses of structures with frequency related loss factor (Jiang *et al.* 2010). Kazemirad *et al.* (2013) analyzed the dynamic properties of frequency-dependent damping systems with soft materials. Sun *et al.* (2024b) constructed the complex modal superposition of multi-degree-of-freedom systems with frequency related loss factor. However, the calculation processes of these methods are complex, and these methods are difficult to apply to wood structures.

In this paper, the relationship between loss factor and damping coefficient was determined based on equal dissipated energy. A modal superposition method is proposed to calculate the dynamic response of wood structures with frequency related damping parameter. Moreover, the frequency-domain method based on frequency related loss factor, the equivalent time-domain method based on constant loss factor, and the proposed method are compared and analyzed.

MODAL SUPERPOSITION METHOD OF WOOD STRUCTURES WITH FREQUENCY RELATED DAMPING PARAMETER

Construction of Damping Matrix

The time-domain motion equation of a single-degree-of-freedom system can be expressed according to Nkibeu *et al.* (2024),

$$m\ddot{x}(t)+c\dot{x}(t)+kx(t)=f \tag{1}$$

where m is the mass; c is the damping coefficient; k is the stiffness; and f is the external excitation; $x(t)$ is the displacement; $\dot{x}(t)$ is the velocity; and $\ddot{x}(t)$ is the acceleration.

The displacement response of a single-degree-of-freedom system under harmonic action is:

$$x(t)=A\sin\theta t \tag{2}$$

where A is the amplitude of the structural displacement response and θ is the vibration frequency of the external excitation harmonic.

Under the influence of harmonics, the energy dissipated per cycle in a single-degree-of-freedom system based on the viscous damping model is (Clough and Penzien 1993):

$$\Delta E=\int_0^{\frac{2\pi}{\theta}} c\dot{x}(t)dx(t) \tag{3}$$

Substituting Eq. 2 into Eq. 3:

$$\Delta E=\pi c\theta A^2 \tag{4}$$

In the process of harmonic vibration, the maximum potential energy within one cycle is (Clough and Penzien 1993):

$$U=\frac{1}{2}kA^2 \tag{5}$$

The loss factor of dissipated energy is (Li *et al.* 2025):

$$\eta=\frac{\Delta E}{2\pi U} \tag{6}$$

The dissipated energy based on the loss factor is:

$$\Delta E=\pi\eta kA^2 \tag{7}$$

Eq. 4 is then set equal to Eq. 7, namely:

$$c=\frac{\eta k}{\theta} \tag{8}$$

The relationship between loss factor and damping coefficient is shown in Eq. 8. The vibration frequency can be further expanded from the external excitation frequency to the entire frequency domain (Bert 1973), Eq. 8 can be rewritten as,

$$c = \frac{\eta}{\varpi} k \tag{9}$$

where ϖ is the structural vibration frequency.

However, the wood loss factor is a function related to the vibration frequency (Jiang *et al.* 2010). The constant loss factor can be replaced with loss factor function (McDaniel *et al.* 2000). Equation 9 can be further expressed as:

$$c = \frac{\eta(\varpi)}{\varpi} k \tag{10}$$

The expression for the damping matrix can be further obtained from Eq. 10, which is:

$$\mathbf{C} = \frac{\eta(\varpi)}{\varpi} \mathbf{K} \tag{11}$$

MODAL SUPERPOSITION METHOD

The time-domain motion equation of multi-degree-of-freedom systems for wood structures is,

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f} \tag{12}$$

where $\mathbf{M}$ is mass matrix; $\mathbf{K}$ is stiffness matrix; $\mathbf{C}$ is damping matrix; $\mathbf{x}(t)$ is structural displacement vector; and $\mathbf{f}$ is the vector of external excitation.

The modal vector of Eq. 12 is:

$$\boldsymbol{\phi}_l = [\boldsymbol{\phi}_1 \quad \boldsymbol{\phi}_2 \quad \cdots \quad \boldsymbol{\phi}_N] \tag{13}$$

The damping matrix $\mathbf{x}(t)$ can be linearly expressed by the complex mode vector, namely:

$$\mathbf{x}(t) = \sum_{n=1}^{N} q_n(t)\boldsymbol{\phi}_n \tag{14}$$

Equation 14 is substituted into Eq. 12, which can be decoupled into N single degree of freedom equations. The single degree of freedom equation of the n-th vibration mode can be expressed as:

$$\ddot{q}_n(t) + \frac{\eta(\varpi)}{\varpi}\omega_n^2 \dot{q}_n(t) + \omega_n^2 q_n(t) = \delta_n \tag{15}$$

where,

$$m_n = \boldsymbol{\phi}_n^{\mathrm{T}} \mathbf{M} \boldsymbol{\phi}_n \tag{16}$$

$$k_n = \boldsymbol{\phi}_n^{\mathrm{T}} \mathbf{K} \boldsymbol{\phi}_n \tag{17}$$

$$\delta_n = \frac{\boldsymbol{\phi}_n^{\mathrm{T}} \mathbf{f}}{\boldsymbol{\phi}_n^{\mathrm{T}} \mathbf{M} \boldsymbol{\phi}_n} \tag{18}$$

$$\omega_n = \sqrt{\frac{k_n}{m_n}} \tag{19}$$

and ω_n is the natural frequency of the n-th vibration mode.

It is assumed that,

$$q_n(t) = q_{n,\mathrm{t}}(t) + q_{n,\mathrm{s}}(t) \tag{20}$$

where $q_{n,\mathrm{t}}(t)$ is the general solution of homogeneous equation of Eq. 12; $q_{n,\mathrm{s}}(t)$ is the special solution of non-homogeneous equation of Eq. 12.

Eq. 20 is substituted into Eq. 15, namely:

$$\ddot{q}_{n,\mathrm{t}}(t) + \frac{\eta(\varpi)}{\varpi} \omega_n^2 \dot{q}_{n,\mathrm{t}}(t) + \omega_n^2 q_{n,\mathrm{t}}(t) = 0 \tag{21}$$

$$\ddot{q}_{n,\mathrm{s}}(t) + \frac{\eta(\varpi)}{\varpi} \omega_n^2 \dot{q}_{n,\mathrm{s}}(t) + \omega_n^2 q_{n,\mathrm{s}}(t) = \delta_n \tag{22}$$

For Eq. 21, the vibration frequency is approximately equal to the natural frequency. Eq. 21 can be rewritten as:

$$\ddot{q}_{n,\mathrm{t}}(t) + \frac{\eta(\omega_n)}{\omega_n} \omega_n^2 \dot{q}_{n,\mathrm{t}}(t) + \omega_n^2 q_{n,\mathrm{t}}(t) = 0 \tag{23}$$

To solve Eq. 23, the time is discretized as:

$$t_k = k\Delta t \quad (k = 0, 1, 2, \ldots) \tag{24}$$

Equation 24 is substituted into Eq. 23, and the solution of Eq. 23 is obtained as (Wang *et al.* 2023),

$$q_{n,\mathrm{t}}(t) = \mathrm{e}^{-\alpha_n (t - t_k)} \left[A_1 \cos \omega_{n,\mathrm{d}} (t - t_k) + A_2 \sin \omega_{n,\mathrm{d}} (t - t_k) \right] \tag{25}$$

where:

$$\begin{cases} \alpha_n = \dfrac{1}{2} \eta(\omega_n) \\ \omega_{n,\mathrm{d}} = \omega \sqrt{1 - \dfrac{1}{4} \eta^2(\omega_n)} \end{cases} \tag{26}$$

In Eq. 22, the loss factor is a function of vibration frequency. The time-domain method cannot be directly applied to solve the Eq. 22. Therefore, the frequency-domain method is considered for solving the Eq. 22. Based on Fourier transform method, the frequency-domain expression of Eq. 22 can be obtained as,

$$-\varpi^2 Q_{n,\mathrm{s}}(\varpi) + \mathrm{i}\, \varpi \eta(\varpi) \omega_n^2 Q_{n,\mathrm{s}}(\varpi) + \omega_n^2 = -\Delta_n(\varpi) \tag{27}$$

where $Q_{n,s}(\varpi)$ is the Fourier transform term of $q_{n,s}(t)$; and $\Delta_n(\varpi)$ is Fourier transform term of δ_n.

By aid of frequency-domain method, $\eta(\varpi)$ can be obtained based on discrete frequencies. Then, the frequency-domain solution of Eq. 27 can be obtained. Based on inverse Fourier transform method, $q_{n,s}(t)$ is obtained.

Both $q_{n,t}(t)$ and $q_{n,s}(t)$ are substituted into Eqs. 14 and 20, respectively. Then, $\mathbf{x}(t)$ is obtained and the modal superposition method of wood structures with frequency-dependent damping parameter is realized.

NUMERICAL EXAMPLES

The loss factor of wood material is frequency related. In Jiang *et al.* (2010), the loss factors of wood material at different vibration frequencies can be obtained at 25 °C. The frequency range is 0.1 to 100 Hz. By aid of the least square method, the relationship of loss factor and vibration frequency can be obtained, which is shown in Eq. 28. The comparison is shown in Fig. 1,

$$\begin{aligned}\eta(\varpi) = 3.346\times10^{-12}\varpi^6 - 9.538\times10^{-10}\varpi^5 + 1.031\times10^{-7}\varpi^4 \\ -5.317\times10^{-6}\varpi^3 + 0.0001351\varpi^2 - 0.001502\varpi + 0.01708\end{aligned} \tag{28}$$

where the coefficient of determination R^2 is 0.8986.

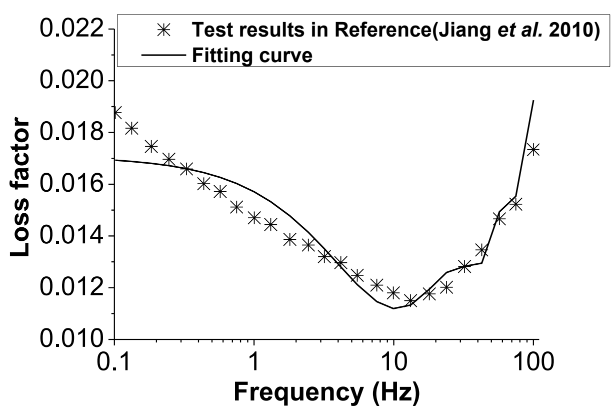

Fig. 1. The dimensions of frame structures and the corresponding load conditions

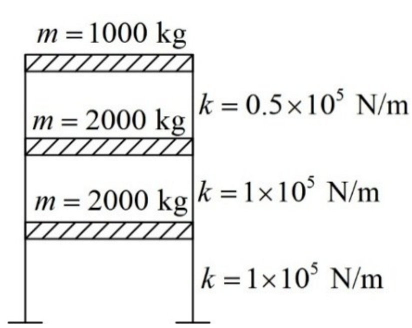

Fig. 2. Schematic diagram of numerical model for the wood structure

The numerical model of wood structure was constructed, which is shown in Fig. 2. The relationship of loss factor and vibration frequency for Eq. 28 was adopted. The natural frequencies of numerical model were 0.5687 Hz, 1.3102 Hz, and 1.9129 Hz.

First, the sine wave was selected as the external excitation. The vibration amplitude was 100 mm/s^2 and the vibration frequency was 30 rad/s. The proposed method (frequency-related time domain, FRTD) and the frequency-domain method (FD) (Clough and Penzien 1993) were adopted, respectively. The corresponding time-domain responses are shown in Fig. 3. The results show that after the time reached 120 s, the calculation results of the two methods were approximately equal. The correctness of the proposed method was indirectly proven. Moreover, the traditional frequency-domain method cannot consider the influence of the transient vibration response. Compared with the frequency-domain method, the proposed method can be used to calculate the transient vibration responses.

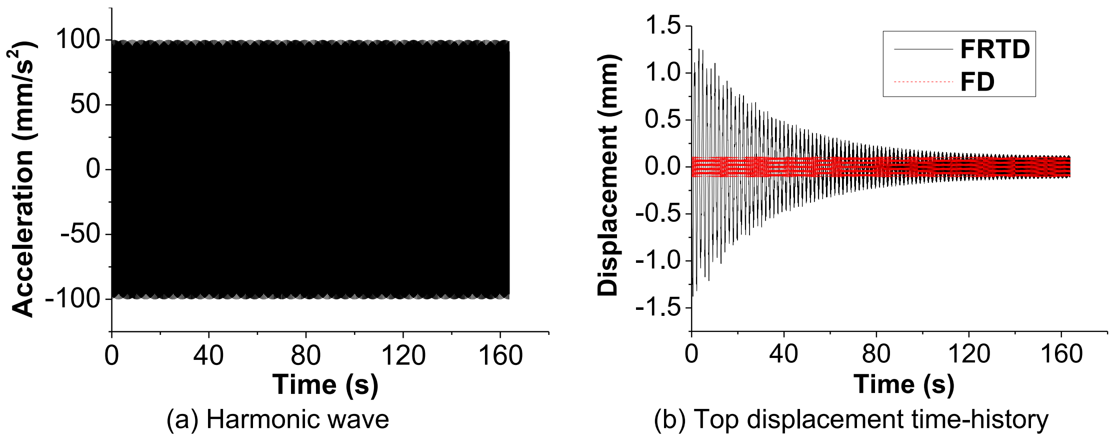

Fig. 3. Time-domain dynamic responses under harmonic wave

The El Centro earthquake wave was selected as the external excitation, and the acceleration time-history is shown in Fig. 4a. The two time-domain methods can be used to calculate the displacement time-history of the numerical model, and the top displacement time-history is shown in Fig. 4b.

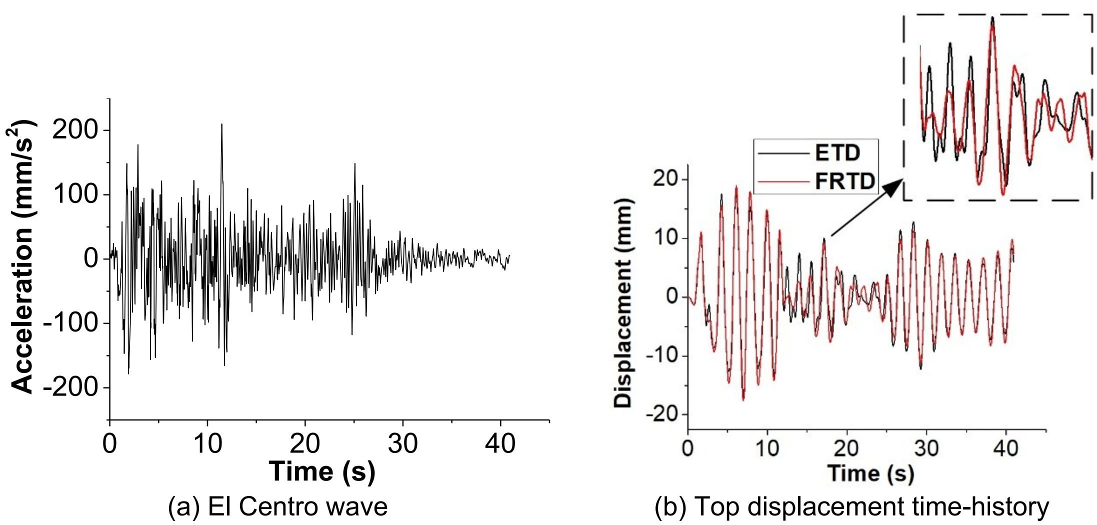

Fig. 4. Time-domain dynamic responses under El Centro wave

One is the equivalent time-domain method (ETD) based on constant loss factor. ETD is the traditional modal superposition method (Clough and Penzien 1993). The constant loss factor is the average value of three modal loss factors, which is 0.015. The other is the proposed method (frequency-related time domain, FRTD). Figure 4b shows that there were local differences in the calculation results between the two methods. Moreover, the peak displacement of ETD was 17.4975 mm, and the peak displacement of FRTD was 18.9338 mm. The relative difference was 8.21%, which is greater than 5%. The reason is that ETD cannot consider the influence of frequency related loss factor. The proposed method based on frequency related loss factor should be preferred to calculate dynamic responses of wood structures.

CONCLUSIONS

1. Based on the principle of equal dissipated energy, the relationship between loss factor and damping coefficient was determined. Then, the damping matrix of wood structures with frequency related damping parameter was established. A combination of time-domain and frequency-domain methods was proposed to solve the dynamic response of wood structures.
2. Compared with the frequency-domain method, the proposed method can additionally consider the transient vibration responses of wood structures. Compared with the equivalent time-domain method based on constant loss factor, the proposed method can additionally consider the influence of frequency related loss factor. The proposed method based on frequency related loss factor should be preferred to calculate dynamic responses of wood structures.

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