

Review on Numerical Homogenization of Corrugated Materials. State-of-the-art in Modeling of Corrugated Board

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Corrugated materials, particularly corrugated board, form the backbone of contemporary packaging due to their light weight and high-strength properties. The application of numerical homogenization techniques to model and predict the mechanical behavior of these materials has evolved significantly, enabling refined structural design and optimization. This review examines advances in the homogenization of corrugated structures, with an emphasis on analytical, numerical, and experimental approaches as applied to corrugated board. Developments in the theoretical modeling of key mechanical properties, such as elasticity, bending, and shear stiffness, are highlighted, alongside methods for predicting structural responses under varying loading conditions. Efforts to optimize structural design through homogenization and the integration of digital tools, including artificial intelligence, are also discussed. Additionally, challenges in adapting homogenization models to account for environmental factors such as humidity and temperature, which impact mechanical properties, are analyzed. The review concludes by outlining future research directions and opportunities for bridging theoretical advances with practical applications in corrugated material design and usage.

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INTRODUCTION

Background and Importance of Corrugated Board

Corrugated board, a staple material in packaging, is celebrated for its excellent combination of mechanical properties, sustainability, and cost-efficiency. Its layered structure, consisting of a corrugated core between flat liners, provides exceptional stiffness and strength while maintaining low weight (Shaban and Alibeigloo 2020; Biancolini and Brutti 2003). This unique combination makes it indispensable in industries that demand both high performance and environmental responsibility (Hietala *et al.* 2016; Hoai *et al.* 2024). The geometry of the corrugated core plays a pivotal role in determining the overall mechanical properties of the material (Basaruddin *et al.* 2014). For instance, the shape of the fluting—whether sinusoidal, trapezoidal, or other forms—directly influences load distribution and energy absorption (Tian *et al.* 2021; Zhuang *et al.* 2019).

Mechanical Properties and Structural Behavior

The mechanical properties of cardboard, such as bending stiffness (Hoon 2024; Mrówczyński *et al.* 2023; Garbowski *et al.* 2022), torsional stiffness (Dang *et al.* 2024; Gajewski *et al.* 2021), transverse shear stiffness (Garbowski and Gajewski 2021; Jelovica and Romanoff 2020), and compressive strength (Garbowski *et al.* 2021; Gajewski *et al.* 2021), are directly influenced by the geometry of the core and the material characteristics of the liners (Aboura *et al.* 2004; Hammou *et al.* 2011). Numerical homogenization methods have proven essential for predicting these macroscopic properties based on the microstructure, enabling more efficient and reliable designs (Marek and Garbowski 2014; Garbowski and Jarmuszcak 2014c; Garbowski and Jarmuszcak 2014d; Bartolozzi *et al.* 2014; Pydah and Batra 2018). The term “homogenization,” as used in this article, refers to the process of replacing the detailed microstructure of corrugated materials with an equivalent continuum model that preserves their macroscopic mechanical behavior, enabling efficient numerical simulation and structural analysis. Numerical simulations (Garbowski and Jarmuszcak 2014a; Garbowski and Jarmuszcak 2014b), combined with experimental validation (Biancolini and Brutti 2003; Czechowski *et al.* 2021), have further enhanced our understanding of the stress distribution and deformation behavior under various loading conditions (Bartolozzi *et al.* 2015; Zou *et al.* 2023).

Advances in Hybrid and Hierarchical Structures

Efforts to improve the mechanical performance of corrugated board have led to the development of hybrid and hierarchical structures. For example, combining honeycomb and corrugated configurations has been shown to significantly enhance stiffness and strength while maintaining lightweight properties (Khatib *et al.* 2024; Shamsi *et al.* 2024). Moreover, optimization studies have explored the structural potential of corrugated cores for applications beyond packaging, such as in aerospace and automotive sectors (Mohammadabadi *et al.* 2019; Kang *et al.* 2022).

The transverse shear stiffness of corrugated cardboard is a critical parameter, especially for structural performance under shear loading. Recent studies have provided robust analytical and numerical frameworks to accurately calculate this property, offering valuable tools for the design of advanced corrugated structures (Garbowski and Gajewski 2021). Similarly, experimental investigations have demonstrated the significant impact of morphological parameters, such as flute height and wavelength, on the mechanical behavior of the material (Huang *et al.* 2022; Amrani *et al.* 2022).

Environmental Factors Affecting Mechanical Performance

Environmental factors, such as humidity and temperature (Mrówczyński *et al.* 2024), significantly impact the performance of corrugated materials, which necessitates robust models for accurate predictions (Amrani *et al.* 2022; Smardzewski and Jasińska, 2017). Recent studies have also explored novel approaches to strengthening corrugated layers, such as chemical treatments and hybrid core designs, which open new avenues for performance enhancement (Hietala *et al.* 2016; Zhu *et al.* 2022). Elevated humidity levels can weaken the core and liners, leading to reduced stiffness and strength. Studies have shown that incorporating advanced coatings or selecting materials with higher moisture resistance can mitigate these effects (Cornaggia *et al.* 2023; Dayyani *et al.* 2013). Khan and Chakraborty (2025) extended this work by integrating thermomechanical properties into homogenization frameworks, allowing for more resilient packaging designs. Similarly,

dynamic analyses by Dharmasena *et al.* (2009) revealed that the layered structure's response to high-frequency loads is sensitive to changes in environmental conditions.

Shi *et al.* (2021) developed models to predict the high-temperature behavior of corrugated composites (Denes *et al.* 2017), providing valuable insights for designing materials capable of withstanding extreme conditions. Gu *et al.* (2017) highlighted the combined effects of moisture and temperature on dynamic performance, showing that transient exposure to such conditions can lead to non-linear deformation and decreased stability. These findings underscore the importance of incorporating environmental factors into homogenization models and material optimization strategies to ensure robust performance across various applications. Furthermore, understanding the nonlinear buckling behavior (Johnson *et al.* 2019; Nam *et al.* 2024) of corrugated cores under varying environmental conditions has been essential for optimizing their performance in diverse applications (Dang *et al.* 2024; Torabi and Niiranen, 2021).

Modeling Approaches for Corrugated Materials

The development of analytical, numerical, and experimental methods has not only advanced the understanding of corrugated cardboard but has also provided insights into optimizing its design for diverse applications (Hashemi-Karouei *et al.* 2022; Suarez *et al.* 2021a). For instance, numerical models such as finite element analysis (FEA) have enabled detailed simulations of the material's response under dynamic conditions, such as drop tests (Hammou *et al.* 2011; Dayyani *et al.* 2013). Similarly, experimental investigations continue to validate and refine these models, ensuring their applicability to real-world scenarios (Tien *et al.* 2023; Garbowski and Jarmuszcak 2014a; Garbowski and Jarmuszcak 2014b).

Finally, the evaluation of equivalent stiffness properties remains a key focus in the field, providing critical insights into how the geometric and material characteristics of the core influence overall structural behavior. Such models have laid the groundwork for future innovations in corrugated materials, enabling their use in increasingly demanding applications (Biancolini 2005).

Corrugated cardboard, as a layered composite material, presents a significant challenge in accurately predicting its macroscopic properties due to its intricate microstructure. Homogenization methods have become an indispensable tool in understanding and optimizing the mechanical behavior of such materials. The primary objective of this work is to provide a comprehensive review of these methods, focusing on their application to corrugated cardboard and highlighting their strengths and limitations.

Analytical homogenization approaches have been widely used to model the effective properties of corrugated materials. These methods rely on simplifying assumptions about the geometry and material properties to derive closed-form solutions. For instance, Abbès and Guo (2010) developed an analytical model to evaluate the torsional stiffness of orthotropic layers in corrugated structures, providing a foundational framework for homogenization in such materials. Similarly, Talbi *et al.* (2009) proposed an analytical approach to capture finite deformation effects in corrugated cores, extending the applicability of homogenization to more complex loading scenarios.

Pydah and Batra (2018) introduced an analytical solution for cylindrical bending, focusing on the impact of geometric parameters such as flute height and wavelength on bending stiffness. These studies demonstrate the versatility of analytical methods in providing insights into the structural behavior of corrugated cardboard under idealized

conditions. However, their reliance on simplifying assumptions often limits their accuracy for real-world applications, especially under non-linear or dynamic loading (Vu *et al.* 2024).

Numerical homogenization methods, particularly those based on the finite element method (FEM), have emerged as powerful tools for studying corrugated materials. These methods enable the detailed modeling of microstructures and their interaction under various loading conditions. Hammou *et al.* (2012) employed a finite element-based homogenization approach to simulate the elastic-plastic behavior of corrugated cores, demonstrating the method's potential for accurately predicting mechanical properties.

Recent advancements in numerical techniques have further improved the accuracy and efficiency of homogenization models. Mrówczyński *et al.* (2022b) utilized FEM to perform numerical homogenization on single-walled corrugated cardboard, capturing the anisotropic behavior of the material. Similarly, Garbowski *et al.* (2021) focused on simulating the crushing behavior of corrugated layers, providing insights into the material's failure mechanisms under compressive loads. These studies highlight the ability of numerical methods to handle complex geometries and non-linear material behavior, making them a preferred choice for advanced simulations.

The validity of analytical and numerical models often depends on experimental validation. Bartolozzi *et al.* (2015) conducted static and dynamic tests on corrugated cores to validate numerical homogenization models, confirming their accuracy in predicting mechanical properties. Hybrid approaches that combine experimental data with numerical models have also gained traction. For instance, Luong *et al.* (2021a) integrated elastoplastic behavior from experimental tests into a finite element homogenization framework, enhancing the model's predictive capability.

Additionally, Zhou and Chen (2009) proposed a computation method to derive equivalent elastic constants for corrugated cores, blending analytical solutions with experimental observations. These hybrid approaches bridge the gap between theory and practice, ensuring that homogenization models are both robust and applicable.

Homogenization methods have found diverse applications in the design and analysis of corrugated materials. Di Russo *et al.* (2024) demonstrated their utility in optimizing corrugated cardboard trays, improving both structural performance and material efficiency. Soliman and Kapania (2013) compared various cellular core structures using homogenization techniques, highlighting the advantages of corrugated designs in specific applications.

Emerging trends in homogenization research include multi-scale modeling and the integration of machine learning algorithms. These approaches aim to reduce computational costs while maintaining high accuracy. For example, Buannic *et al.* (2003) explored homogenization of corrugated sandwich panels at multiple scales, providing insights into their global mechanical response.

Despite their success, homogenization methods face several challenges. Analytical models often struggle to account for complex geometries and material anisotropy, while numerical methods can be computationally intensive. Experimental validation, though crucial, is resource-intensive and requires precise measurement techniques (Bartolozzi *et al.* 2015; Vu *et al.* 2024).

Scope and Objective of the Review

This work highlights the importance of corrugated cardboard as a layered material with highly adaptable mechanical properties. By synthesizing insights from diverse methodologies, the goal is to provide a comprehensive understanding of its mechanical behavior and the role of numerical homogenization techniques in optimizing its performance (Buannic *et al.* 2003; Di Russo *et al.* 2024). Although the literature already includes mentions of using Artificial Neural Networks (ANN) in the optimization and load-bearing capacity estimation of corrugated board packaging (Gu *et al.* 2023; Gajewski *et al.* 2024), future research should focus on developing more efficient multi-scale homogenization frameworks, integrating advanced computational tools such as artificial neural networks (*e.g.* Garbowski 2024a), and expanding the application of these methods to novel materials. This work aims to synthesize current knowledge and provide a foundation for advancing the field of homogenization in corrugated cardboard and related materials.

This review is structured as follows: Section 2 introduces the layered structure and fluting shapes of corrugated board. Section 3 discusses mechanical properties. Section 4 highlights the importance of anisotropy. Section 5 presents homogenization methods (analytical, numerical, and experimental). Section 6 explores applications of homogenization, and Section 7 compares approaches for sandwich panels. Section 8 discusses future trends and conclusions.

CHARACTERISTICS OF CORRUGATED BOARD

Layered Structure and Fluting Shapes

As was already mentioned, the geometry of the corrugated core, often referred to as the flute, plays a critical role in determining the overall performance of the structure (Huang *et al.* 2022; Bartolozzi *et al.* 2014). The most common fluting shape in corrugated cardboard is sinusoidal, which is valued for its efficient load distribution and ease of manufacturing. This geometry provides a balance between structural performance and material usage (Bartolozzi *et al.* 2013; Meng *et al.* 2020). However, alternative geometries, such as trapezoidal and rectangular fluting (Garbowski and Borecki 2024; Garbowski, 2025), have been increasingly explored to enhance specific mechanical properties. Trapezoidal cores, for example, offer greater compressive strength due to their ability to reduce stress concentrations, making them suitable for high-load applications (Yang *et al.* 2019; Hazman *et al.* 2017).

Rectangular fluting, although less common, provides enhanced stiffness at the cost of increased material usage. Studies by Meng *et al.* (2020) and Reinaldo Goncalves *et al.* (2016) have shown that rectangular and other hybrid shapes can outperform sinusoidal geometries in specific scenarios, such as dynamic loading or thermal insulation. These findings underscore the potential for optimizing flute shapes to meet diverse performance criteria.

The multi-layered structure of corrugated cardboard enhances its ability to resist bending and compressive loads. The interplay between the core geometry and the liners is crucial in defining the overall stiffness and strength of the material (Hazman *et al.* 2017; Luong *et al.* 2021b). Experimental testing combined with numerical models has

demonstrated the significant influence of core height, flute wavelength, and liner thickness on mechanical performance (Kl̇aslan *et al.* 2014; Huang *et al.* 2022).

Moreover, the layered structure allows for shape optimization to meet specific functional requirements. Laszczyk *et al.* (2010) used a level-set method to optimize the geometry of corrugated cores, achieving significant improvements in stiffness and load-bearing capacity. These studies emphasize the importance of integrating structural design and material selection to maximize performance.

Recent advancements in modeling techniques have expanded our understanding of fluting shapes and their impact on performance (Garbowski 2024, 2025). Finite element models, such as those by Dayyani *et al.* (2013) and Luong *et al.* (2021b), have been instrumental in simulating complex geometries and predicting the mechanical behavior of corrugated cores under various loading conditions. These methods have also facilitated the exploration of novel flute shapes, including hybrid and multi-layer configurations, which have shown promise in applications requiring high strength-to-weight ratios (Dharmasena *et al.* 2009; Bartolozzi *et al.* 2013).

Additionally, thermomechanical analyses have revealed the potential for integrating corrugated structures into environments with varying thermal and mechanical demands (Khan and Chakraborty 2021; Takano and Zako 1995). These findings highlight the versatility of corrugated cores and their potential applications beyond traditional packaging, such as in aerospace and construction.

Despite significant progress, challenges remain in optimizing flute shapes for specific applications. Manufacturing constraints and material limitations often dictate the choice of core geometry, limiting the exploration of unconventional designs (Meng *et al.* 2020; Kl̇aslan *et al.* 2014). However, advancements in computational tools and experimental techniques continue to open new avenues for innovation. For instance, the integration of shape optimization algorithms with material science has the potential to redefine the performance boundaries of corrugated materials (Laszczyk *et al.* 2010; Huang *et al.* 2022).

In summary, the layered structure and diverse fluting geometries of corrugated cardboard offer unparalleled opportunities for tailored design. By leveraging advanced modeling techniques and exploring alternative geometries, researchers and engineers can unlock the full potential of this versatile material.

Mechanical Properties

Corrugated cardboard's performance as a structural material is largely determined by its bending stiffness and compressive strength. These mechanical properties are crucial for applications such as packaging, where the material must withstand both static and dynamic loads. Understanding the factors influencing these properties is essential for optimizing the design and functionality of corrugated structures (Al Hemeiri *et al.* 2020; Cornaggia *et al.* 2023).

Bending stiffness is a key parameter that defines the resistance of corrugated cardboard to deformation under flexural loads. It is primarily influenced by the geometric configuration of the corrugated core and the material properties of the liners. Meng *et al.* (2020) demonstrated that rectangular fluting shapes significantly enhance bending stiffness compared to sinusoidal configurations, though at the cost of increased material usage. Analytical and numerical models have been developed to predict bending stiffness based on these parameters, providing reliable tools for material design (Gajewski *et al.* 2021).

Compressive strength defines the material's ability to resist crushing under axial loads, a critical property for applications involving stacking or storage. The interaction between the corrugated core and the liners significantly affects this property. Han *et al.* (2017) explored foam-filled composite corrugated cores and demonstrated a substantial increase in compressive strength, highlighting the potential for hybrid structures in high-load applications.

The geometry of the fluting also plays a critical role. Rectangular and trapezoidal cores exhibit higher compressive strengths compared to sinusoidal shapes due to their ability to distribute loads more evenly (Vu *et al.* 2024). However, this improvement is often accompanied by higher production costs and material demands.

Advanced modeling techniques have been instrumental in analyzing and optimizing bending stiffness and compressive strength. Al Hemeiri *et al.* (2020) used homogenization methods to derive in-plane moduli of elasticity, allowing for accurate predictions of mechanical behavior under varying conditions. Gajewski *et al.* (2021) combined numerical simulations with experimental tests to investigate the crushing behavior of double-walled corrugated boards, providing insights into failure mechanisms and potential design improvements.

Hybrid models that integrate experimental data with analytical and numerical methods are increasingly used to validate theoretical predictions. For instance, Dharmasena *et al.* (2009) employed dynamic response tests to corroborate the results of finite element simulations, ensuring the reliability of their findings.

Despite significant advancements, challenges remain in accurately predicting the mechanical properties of corrugated cardboard under real-world conditions. The interaction between geometric and material variables, combined with environmental influences, introduces complexities that require robust multi-scale models. Future research should focus on integrating machine learning techniques with existing modeling frameworks to enhance predictive accuracy and computational efficiency (Han *et al.* 2017; Cornaggia *et al.* 2023).

In summary, bending stiffness and compressive strength are pivotal to the performance of corrugated cardboard. By leveraging advanced modeling techniques and exploring innovative structural configurations, researchers can continue to optimize these properties for a wide range of applications.

Influence of Anisotropy

Corrugated cardboard, as a multilayer material, exhibits significant anisotropy due to its structure and the properties of its components, such as the paper used in the outer and corrugated inner layers (Dang *et al.* 2024). Anisotropy plays a critical role in the homogenization process, as the mechanical properties of the material vary depending on the loading direction. It influences stiffness, strength, and deformation behavior, all of which are essential in modeling and optimizing the performance of corrugated cardboard under various load conditions. Understanding and incorporating anisotropy into constitutive models is fundamental for developing effective homogenization techniques. This section presents the evolution of constitutive models for anisotropic materials such as paper and cardboard. A chronological review of key works highlights how advancements in these models have improved the ability to accurately represent the behavior of fibrous materials and their role in the homogenization process. These models not only account for the directional differences in material behavior but also lay the groundwork for more

precise predictions of mechanical performance, which is critical for the effective design and optimization of corrugated cardboard structures.

Hill (1948) laid the groundwork for modeling anisotropic materials by proposing a yield criterion that extended the Huber-Mises criterion to account for anisotropy through six parameters. This approach, designed for metals, provided a basis for understanding the yielding and plastic flow of materials with preferred orientations and was later adapted for other anisotropic materials, including paper and paperboard. Hoffman (1967) introduced a phenomenological fracture condition for orthotropic brittle materials, emphasizing the differences in tensile and compressive strengths along various directions. This work highlighted the need for anisotropic models that account for directional variations, influencing subsequent developments in constitutive modeling.

Tsai and Wu (1971) proposed an operationally simple strength criterion for anisotropic materials. Unlike earlier quadratic approximations, this model satisfied coordinate transformation invariance and incorporated independent interaction terms, enabling predictions for multi-axial stress states. This method became widely applied to composite materials, including fibrous structures such as paper. Xia *et al.* (2002) developed a three-dimensional anisotropic constitutive model for paper and paperboard, directly constructing the initial yield surface from experimental data. By incorporating anisotropic strain hardening and a procedure to identify material properties, the model accurately captured the elastic-plastic deformation of paper under various loading conditions.

Mäkelä and Östlund (2003) introduced an orthotropic elastic-plastic model with isotropic hardening for paper. Their work employed uniaxial tensile tests in different in-plane directions to calibrate material parameters. The model was validated numerically and experimentally, demonstrating its capability to describe the nonlinear mechanical behavior of paper. Harrysson and Ristinmaa (2008) extended the modeling of paper by incorporating a multiplicative split of the deformation gradient to introduce anisotropic plasticity. Using the Tsai-Wu criterion for yield and a non-associated plasticity theory, their model was calibrated with biaxial tension tests and applied to simulate creasing processes in corrugated boards.

Borgqvist *et al.* (2014) presented an elasto-plastic model for orthotropic materials, addressing limitations in predicting softening effects under non-proportional loading. By introducing a yield surface with multiple hardening variables, the model captured the behavior of pre-strained paperboard samples. Implementation in finite element frameworks validated the model's predictive capabilities. Michel and Billington (2014) explored anisotropy in biobased composites, proposing a transversely isotropic nonlinear constitutive model. Though focused on natural composites, their work highlighted parallels with paperboard by addressing directional elastic-plastic behavior and calibrating parameters through simple mechanical tests.

Borgqvist *et al.* (2015) refined their earlier model by introducing a continuum framework to account for anisotropic behavior in paperboard during large plastic deformations. Using director vectors for preferred material directions, the model simulated industrial processes such as creasing, providing insights into the anisotropic plasticity of paper. Tjahjanto *et al.* (2015) developed a continuum model for high-density cellulose-based materials, combining viscoelastic and viscoplastic formulations. Their approach incorporated anisotropic hardening and material densification effects, offering a comprehensive framework for simulating complex stress-strain behavior under transient loading conditions.

Li *et al.* (2016) proposed an orthotropic elastic-plastic model for laminated paperboard, incorporating nonlinear kinematic and isotropic hardening. The model reduced the number of required parameters by preserving compressive yield stress during reverse loading, achieving accurate predictions for anisotropic behavior under complex loading conditions. Seidlhofer *et al.* (2021) advanced the understanding of pulp fiber anisotropy by using nanoindentation experiments to capture compressible plastic and nonlinear elastic behavior. Their numerical model, based on a hyper-foam and compressible plasticity framework, achieved good agreement with experimental data and emphasized the importance of nanoporous structures in anisotropic responses.

The development of anisotropic constitutive models for paper and paperboard has progressed significantly over decades, starting with general theories for metals and evolving into sophisticated frameworks tailored for fibrous materials. These models now capture complex behaviors, such as nonlinear elasticity, plasticity, and directional hardening, enabling accurate simulations of industrial processes and material performance under various conditions (Garbowski *et al.* 2012). By understanding the specific material model and the parameters that define it, it becomes possible to tailor appropriate homogenization techniques to characterize these parameters effectively.

For example, advanced constitutive models require inputs such as elastic constants, yield stress, and hardening parameters, which are inherently direction-dependent in anisotropic materials. Homogenization techniques, such as the Representative Volume Element (RVE) approach or continuum-based methods, can be adapted to extract these parameters by averaging microstructural behaviors while preserving the anisotropic nature of the material. Knowing the material model also allows researchers to design experiments or simulations that target specific behaviors, such as in-plane stiffness, bending rigidity, or shear response, which are critical for accurately defining the constitutive parameters.

Additionally, by integrating the material model with homogenization strategies, it is possible to bridge the gap between microstructural characteristics (*e.g.*, fiber orientation, bonding quality) and macroscopic performance. This synergy ensures that homogenized properties not only reflect the material's global behavior but also align with the anisotropic features embedded in the constitutive model. As a result, the combination of advanced material modeling and tailored homogenization techniques provides a robust framework for analyzing and optimizing corrugated cardboard and other anisotropic materials, enhancing their design and application in industrial contexts.

HOMOGENIZATION METHODS

Analytical Approaches

Analytical methods play a fundamental role in the homogenization of corrugated materials, offering closed-form solutions that describe the effective properties of these complex structures. These approaches are particularly valued for their ability to provide theoretical insights and establish a foundation for numerical and experimental methods.

Analytical homogenization relies on simplifying assumptions about the geometry and material properties of corrugated layers to derive equivalent macroscopic properties. Abbès and Guo (2010) proposed an analytical model for determining the torsional stiffness of orthotropic layers, which has become a benchmark in this domain. Similarly, Talbi *et*

al. (2009) developed a homogenization model to address finite deformations in layered materials, expanding its applicability to real-world scenarios.

Pydah and Batra (2018) presented a closed-form solution for cylindrical bending, focusing on the interplay between flute geometry and bending stiffness. These models enable rapid calculations of effective properties and are particularly useful in preliminary design phases (Buannic *et al.* 2003; Zhou and Chen, 2009).

The versatility of analytical models has been demonstrated in various applications, from corrugated cardboard to sandwich panels. For instance, Duong *et al.* (2013) introduced a model for coupling shear and torsion effects in corrugated layers, providing valuable insights for load-bearing applications. Similarly, Mohammadabadi *et al.* (2019) applied analytical methods to wood-composite sandwich panels with corrugated cores, illustrating their utility in diverse materials.

In packaging applications, analytical methods have been used to predict the transverse shear stiffness of corrugated boards. Garbowski and Gajewski (2021) employed such models to analyze single and multi-layered structures, offering design recommendations for improved mechanical performance.

Analytical methods are celebrated for their simplicity and computational efficiency. Sharma *et al.* (2010) highlighted their effectiveness in predicting the macroscopic behavior of plates with microstructures, enabling engineers to optimize designs without the computational cost of numerical simulations. However, these models often rely on assumptions that limit their accuracy for highly non-linear or anisotropic materials, as demonstrated by Vu *et al.* (2024).

Takano and Zako (1995b) underscored the challenges in extending analytical homogenization to multi-scale problems, where the interaction between layers and complex geometries requires more detailed modeling. These limitations suggest a need for hybrid approaches that integrate analytical methods with numerical or experimental techniques.

Recent advancements have focused on enhancing the accuracy and applicability of analytical models. Hammou *et al.* (2011) integrated experimental data to validate their analytical predictions for drop tests on corrugated materials. Such hybrid approaches are expected to bridge the gap between theoretical simplicity and practical reliability.

Moreover, Xing *et al.* (2023) proposed incorporating machine learning algorithms into analytical frameworks, allowing for real-time predictions of effective properties based on geometric and material inputs. This direction holds promise for expanding the use of analytical homogenization in automated and high-throughput design processes.

Analytical methods remain a cornerstone of homogenization techniques for corrugated materials, providing essential tools for understanding and optimizing their mechanical behavior. While challenges persist, particularly in handling non-linearities and complex interactions, ongoing advancements in hybrid methods and computational integration promise to extend their utility in both research and industry.

Numerical Approaches

Numerical methods are indispensable tools in the homogenization of corrugated materials, enabling detailed simulations of their complex structures and behaviors. Among these, the finite element method (FEM) and multi-scale modeling have emerged as the most prominent techniques, offering insights into mechanical properties and structural responses under various conditions.

The finite element method has been extensively used to model the mechanical behavior of corrugated cardboard, providing high accuracy in predicting stiffness, strength, and failure mechanisms. Aduke *et al.* (2024) demonstrated the application of FEM to simulate bending and compression in corrugated paperboard, capturing the anisotropic properties of the material. Similarly, Garbowski *et al.* (2021) used FEM to determine the transverse shear stiffness of single and multi-layered corrugated structures, offering practical insights for packaging design.

One of the key advantages of FEM lies in its ability to simulate the interaction between layers in corrugated boards. Luong *et al.* (2021a) developed an elastoplastic FEM model to analyze the deformation behavior under varying loading conditions, while Hammou *et al.* (2012) applied FEM to study elastic-plastic transitions in corrugated cores. These studies highlight FEM's versatility in addressing both linear and non-linear behaviors.

The method has also proven effective in failure analysis. Mrówczyński *et al.* (2022a) employed FEM to predict failure mechanisms in single-walled corrugated boards under compressive loads, providing valuable data for material optimization. Similarly, Garbowski and Borysiewicz (2014) simulated the strength of corrugated boards under axial compression, identifying critical failure points and offering strategies for improvement.

Multi-scale modeling complements FEM by bridging the gap between microstructural details and macroscopic behaviors. Basaruddin *et al.* (2014) conducted a stochastic multi-scale analysis to evaluate the homogenized properties of corrugated materials, demonstrating the impact of geometric variability on mechanical performance. These models provide a more holistic view of material behavior by considering both local and global responses.

Cornaggia *et al.* (2023) combined experimental data with multi-scale numerical modeling to study the effects of humidity and temperature on corrugated cardboard. Their approach highlighted the importance of environmental factors in determining the effective properties of the material. Similarly, Kılçaslan *et al.* (2014) used multi-scale methods to link microstructural characteristics of corrugated cores with their macroscopic performance under dynamic loads.

The integration of multi-scale approaches with FEM has further enhanced predictive capabilities. Soliman and Kapania (2013) compared various cellular core geometries using a combined FEM and multi-scale framework, demonstrating the strengths of corrugated designs in load distribution. Zhu *et al.* (2021) extended this approach to composite corrugated cores, revealing their potential for high-load applications.

Multi-scale models have also been instrumental in dynamic analysis. Yang *et al.* (2019) performed static and dynamic simulations of corrugated core structures, providing insights into their behavior under impact and vibration conditions. Liu *et al.* (2021) incorporated homogenization techniques into multi-scale simulations to predict the behavior of corrugated structures with complex geometries, paving the way for advanced material design.

Numerical approaches, particularly FEM and multi-scale modeling, are critical in advancing the understanding of corrugated materials. FEM excels in detailed simulations of mechanical behavior and failure mechanisms, while multi-scale models provide a comprehensive perspective by integrating microstructural and macroscopic analyses. Together, these methods form a powerful toolkit for optimizing the design and performance of corrugated materials across various applications.

Experimental methods play a crucial role in understanding the mechanical behavior of corrugated materials and validating analytical and numerical models. These methods are instrumental in ensuring the reliability of theoretical predictions and providing insights into the material's response under real-world conditions. This section focuses on two key aspects: the validation of analytical and numerical results and commonly employed laboratory tests such as the Edge Crush Test (ECT) and Box Compression Test (BCT).

Validation of Analytical and Numerical Results

The experimental validation of analytical and numerical models is essential for assessing their accuracy and applicability. Bartolozzi *et al.* (2015) and Luong *et al.* (2018) conducted static and dynamic tests on corrugated cores to validate their numerical models, confirming the reliability of predictions for both load distribution and deformation patterns. Similarly, Garbowski *et al.* (2021) examined the crushing behavior of double-walled corrugated boards, demonstrating a strong correlation between experimental results and numerical simulations.

Biancolini and Brutti (2003) combined experimental testing with numerical models to investigate the mechanical behavior of corrugated cores, offering a comprehensive validation framework. Hammou *et al.* (2011) performed drop tests on corrugated materials, using experimental data to refine their numerical simulations and enhance predictive accuracy. Zhu *et al.* (2021) further highlighted the importance of compression testing for validating the mechanical response of composite corrugated cores, showcasing the interplay between geometry and material properties.

Homogenization through Inverse Analysis

Garbowski and Marek (2014) propose an inverse analysis approach for homogenizing corrugated boards, aimed at improving the efficiency of computer-aided design for packaging. While full structural models provide precise results, they are computationally expensive and impractical for industrial use. By adopting homogenized models, engineers can design packaging efficiently and reliably, provided the effective parameters of the simplified model are accurately calibrated to replicate the behavior of the full model across a range of loading conditions.

The current authors present a calibration method that combines simple experimental tests with numerical homogenization techniques for thin periodic plates. While the homogenization of elastic properties of paperboard is well-documented, this study highlights the challenge of calibrating effective properties in the inelastic region. The proposed method integrates laboratory tests, numerical simulations, and inverse analysis to achieve full calibration of simplified models. Additionally, the study includes sensitivity analyses and simplified case studies under various experimental setups, demonstrating the method's robustness and applicability in industrial contexts. This approach offers a practical solution for balancing accuracy and computational efficiency in the design and analysis of corrugated board structures.

Common Laboratory Tests

Laboratory tests are indispensable tools for characterizing the mechanical properties of corrugated materials. Among these, the Edge Crush Test (ECT) and Box Compression Test (BCT) are standardized, *e.g.*, according to TAPPI T811 and ISO 3037 standards and are the most widely used for evaluating stiffness, strength, and load-bearing

capacity. Garbowski *et al.* (2021c) introduced an enhanced ECT configuration that provided more reliable data for validating numerical models. Additionally, Garbowski and Przybyszewski (2015) analyzed the critical force in ECTs, exploring the sensitivity of results to variations in material properties and geometry.

Klçaslan *et al.* (2014) conducted extensive experimental testing of corrugated cores, focusing on their response to bending and compression. Their work highlighted the importance of testing protocols in capturing the anisotropic behavior of corrugated materials. Zou *et al.* (2023) explored the compressive behavior of concrete-filled corrugated cores, using laboratory data to refine their understanding of structural stability under high loads.

Han *et al.* (2014) analyzed collapse mechanisms in foam-filled corrugated cores, employing compressive tests to identify failure modes. These findings have direct implications for designing more resilient materials. Amrani *et al.* (2022) examined the impact of morphological and thermal properties on the performance of corrugated cardboard, emphasizing the role of environmental conditions in laboratory testing.

Leekitwattana *et al.* (2009) propose a bi-directional corrugated-strip-core steel sandwich plate designed to enhance strength and stiffness in both transverse directions, addressing limitations of conventional unidirectional corrugated-core plates. The study introduces an analytical homogenization model to evaluate the transverse shear stiffness and strength of the proposed design. Results indicate that the bi-directional configuration significantly improves these mechanical properties, demonstrating its potential for applications requiring lightweight yet robust structures. This work highlights the role of homogenization in optimizing the design of advanced sandwich plates for enhanced multi-directional performance.

Experimental approaches provide an essential foundation for validating theoretical models and understanding the real-world behavior of corrugated materials. By combining advanced testing techniques with analytical and numerical frameworks, researchers can ensure the accuracy and reliability of their findings. The integration of experimental data into homogenization and optimization processes remains a critical avenue for advancing the design and application of corrugated materials.

Although homogenization techniques provide efficient and reliable predictions of mechanical behavior, they inevitably introduce simplifications compared to full structural models. Full-scale simulations offer higher accuracy but are significantly more computationally expensive. Homogenization thus represents a practical compromise between computational cost and prediction accuracy, especially valuable in industrial applications.

APPLICATION OF HOMOGENIZATION METHODS IN CORRUGATED CARDBOARD

Optimization of Mechanical Properties in the Context of Packaging

The application of homogenization methods to optimize the mechanical properties of corrugated cardboard is a cornerstone of modern packaging design. By leveraging advanced analytical, numerical, and experimental techniques, researchers have developed methods to enhance the performance of corrugated materials, ensuring durability, efficiency, and sustainability.

Garg *et al.* (2014) use homogenization methods to optimize the geometric parameters of unidirectional corrugated-core sandwich panels for enhanced structural performance. By employing an analytical model based on composite lamination theory, the study evaluates the impact of core undulation, amplitude, and corrugation angle on the panel's effective properties. The optimized design improves bending rigidity and material efficiency (Mohammadabadi *et al.* 2020, 2023), thus demonstrating the value of numerical homogenization techniques in balancing performance and resource use for residential building applications.

Ma *et al.* (2017) propose an optimization procedure for a built-up thermal protection system (TPS) using corrugated core homogenization. The system combines a traditional Integrated TPS (ITPS) with an additional insulation layer attached to its cold surface. While the homogenization model shows deviations from exact temperature predictions, it validates the enhanced insulation capability of the built-up TPS. A two-step design procedure is introduced, leveraging finite element and finite difference methods to optimize insulation thickness and reduce TPS areal density, offering a highly efficient and accurate design for thermal protection applications.

Homogenization methods provide tools to predict and enhance the stiffness, compressive strength, and load-bearing capacity of corrugated boards. Garbowski *et al.* (2021d) demonstrated that optimizing the transverse shear stiffness of corrugated cardboard through numerical homogenization significantly improves its ability to withstand stacking loads, a critical requirement in packaging. Similarly, Shamsi Monsef *et al.* (2024) employed structural optimization techniques to design high-performance composite corrugated cores, achieving enhanced mechanical properties while reducing material usage.

Research on corrugated fibreboard highlights the critical role of homogenization techniques, bending stiffness (BS), and Edge Crush Test (ECT) performance in understanding and optimizing its mechanical properties (Popil 2012). Szewczyk and Głowacki (2014) show how humidity impacts BS and ECT through changes in the material's elastic properties, providing a framework for predicting behavior under varying environmental conditions. Similarly, Garbowski and Knitter-Piątkowska (2022) emphasize the importance of BS in packaging design, demonstrating how small imperfections in thin liners can reduce stiffness by up to 10%, validated through experimental and finite element methods.

Jamsari *et al.* (2020) and Jamsari *et al.* (2019) extend this understanding by examining Corrugated Fiberboard's (CFB's) bending and ECT performance under different orientations and levels of pre-crushing. Their models, combining analytical solutions, equivalent flute models, and finite element analyses, reveal that caliper significantly affects BS, while flute geometry plays a secondary role. The real geometry model excelled in predicting ECT performance but failed to account for severe delamination effects, suggesting the need for further refinement.

Szewczyk and Bieńkowska (2020) and Czechowski *et al.* (2021), demonstrate the potential of numerical and data-driven approaches to optimize BS and ECT for corrugated boards. By combining experimental validation with predictive modeling, these works underline the synergy between homogenization and advanced computational methods in enhancing the design and performance of corrugated structures.

Anastas (1998) underscores the significance of the Edgewise Compression Test (ECT) as a predictor of box compression strength, introducing the simplified McKee

formula to link ECT, box perimeter, and caliper, thus streamlining its practical application. Gilchrist *et al.* (1999) complement this by employing nonlinear finite element models to analyze bending stiffness and ECT performance under in-plane and transverse loads, demonstrating strong correlations with experimental data and addressing moisture-induced effects like warp and creep.

Hoon and Park (2004) further explore the relationship between bending stiffness and compression strength, showing through four-point bending tests that variations in panel stiffness, driven by flute type, significantly affect box stacking strength. Their findings highlight the role of flexural stiffness in optimizing corrugated board performance. Finally, Cash and Frank (2020) examine how span length influences bending stiffness measurements, proposing a method to standardize results across varying spans, thus enhancing the reliability and comparability of stiffness evaluations in corrugated materials. These studies collectively enrich the understanding of mechanical behavior and testing methods for corrugated boards.

Experimental studies have also played a key role in validating these improvements. Bartolozzi *et al.* (2015) conducted static and dynamic tests on corrugated cores, confirming the reliability of homogenization models in predicting mechanical performance. These findings underline the importance of integrating experimental data with homogenization techniques for practical applications.

Homogenization has also enabled the development of innovative packaging solutions. Di Russo *et al.* (2024) utilized advanced homogenization models to design corrugated cardboard trays with optimized stiffness and reduced weight, addressing the dual demands of performance and sustainability. Suarez *et al.* (2021b) highlighted the application of homogenization approaches in improving the mechanical efficiency of corrugated boards, emphasizing their role in reducing material consumption without compromising functionality.

Luong *et al.* (2021b) demonstrated the application of finite element-based homogenization models in the elastoplastic analysis of corrugated structures, paving the way for customized packaging designs tailored to specific load conditions. These advancements showcase the potential of homogenization methods to revolutionize the packaging industry.

The integration of homogenization methods with modern optimization algorithms, such as machine learning, represents a promising direction for further improving packaging design. Mrówczyński *et al.* (2022c) proposed a framework for the optimal design of double-walled corrugated boards, incorporating advanced homogenization techniques to achieve unprecedented performance levels. These efforts highlight the potential for homogenization to meet the growing demands for sustainable, lightweight, and high-performance packaging solutions.

Sandwich Panels: Similarities and Differences in Homogenization Approaches

Sandwich panels, like corrugated board, are layered structures designed to optimize mechanical performance (Pradhan *et al.* 2023; Tumino *et al.* 2014). While their applications often overlap, particularly in lightweight construction and packaging, the approaches to their homogenization reveal both similarities and critical differences (Zhuang *et al.* 2021). These distinctions stem from variations in core geometry, material properties, and structural demands.

Both sandwich panels and corrugated cardboard benefit from homogenization techniques that simplify complex microstructures into equivalent macroscopic properties. Buannic *et al.* (2003) developed foundational methods for homogenizing corrugated cores in sandwich panels, demonstrating their applicability in predicting global mechanical behavior. Similarly, Takano and Zako (1995) applied homogenization to stress analysis in sandwich plates, providing a framework that is directly comparable to the approaches used for corrugated cardboard.

The equivalent material models used for both structures often share common principles. For instance, Shaban and Alibeigloo (2020) performed global bending analyses of sandwich panels with corrugated cores (Pradhan *et al.* 2023), highlighting the transferability of homogenization methods between the two material systems. This similarity is particularly evident in analyses of bending stiffness and transverse shear properties, which are critical for both corrugated and sandwich structures (Reinaldo Goncalves *et al.* 2016; Liu *et al.* 2021).

Despite these similarities, significant differences arise from the structural and material design of sandwich panels compared to corrugated cardboard. Sandwich panels often incorporate higher-density core materials, such as foam or honeycomb, which require different homogenization strategies to account for their isotropic or near-isotropic behavior (Mohammadi *et al.* 2015). In contrast, the anisotropic nature of corrugated cores necessitates more specialized approaches to capture directional dependencies in mechanical properties (Zhuang *et al.* 2019).

The geometric complexity of sandwich panel cores also differentiates their homogenization. Zhu *et al.* (2022) introduced a refined plate theory for bending analysis in corrugated sandwich layers, emphasizing the need for multi-variable models to address intricate geometries. Soliman and Kapania (2013) compared various cellular cores, including corrugated and honeycomb, using homogenization approaches, revealing that the choice of core geometry significantly influences the predictive accuracy of homogenized properties.

Homogenization models for sandwich panels are often validated through experimental and numerical methods (Ravishankar *et al.* 2011), similar to those used for corrugated cardboard. Hammou *et al.* (2011) integrated finite element simulations with homogenization techniques to analyze the mechanical behavior of both corrugated and sandwich cores, demonstrating the versatility of these approaches. Smardzewski and Jasińska (2017) highlighted the importance of experimental data in refining homogenization models, particularly for sandwich panels subjected to complex loading conditions.

The application of homogenization methods in sandwich panels extends beyond structural analysis, encompassing thermal and acoustic properties. For instance, Liu *et al.* (2021) proposed an equivalent homogenization method to optimize sandwich panel designs for multi-functional applications, such as energy absorption and insulation.

Homogenization methods for sandwich panels share foundational principles with those used for corrugated cardboard but diverge in their application due to differences in core geometry and material properties. These methods enable the efficient design and analysis of sandwich structures, providing critical insights into their mechanical, thermal, and functional behavior. By exploring both the similarities and differences in homogenization approaches, researchers can tailor these techniques to meet the specific demands of each material system.

Specific Applications of Various Homogenization Techniques

Homogenization techniques provide versatile tools for modeling and optimizing the mechanical performance of corrugated materials and composite structures. Beck and Fischerauer (2022) use Kirchhoff plate theory to homogenize material properties of corrugated cardboard, enabling practical warp analysis and optimization while separating warp from surface irregularities such as washboarding. Similarly, Cheon and Kim (2015) derive stiffness matrices for corrugated-core sandwich panels using classical lamination theory (CLT), simplifying the analysis of tensile and bending behaviors.

Dao *et al.* (2025) streamline computational efficiency through a 2D finite element model using homogenized elastic-plastic properties, significantly reducing analysis time compared to 3D models. Duong (2017) applies analytical homogenization to double corrugated cardboard plates, simplifying 3D structures into 2D equivalents for efficient and accurate bending and shear force simulations.

Debnath *et al.* (2025) extend homogenization beyond structural analysis, demonstrating its role in enhancing recycled fibers from old corrugated containers (OCC) by improving strength and flexibility through high-shear processing. Szewczyk and Głowacki (2014) further explore environmental effects, using elastic material modeling to predict reductions in stiffness and strength due to humidity variations.

Advanced homogenization approaches also address specialized applications. Khan and Chakraborty (2025) propose a thermomechanical homogenization model for Integrated Thermal Protection Systems, accounting for transverse shear and thermal effects. Mrówczyński and Garbowski (2023) include geometric imperfections in homogenization models for multilayer corrugated boards, improving stiffness predictions by accounting for manufacturing flaws.

Experimental and numerical studies complement theoretical homogenization. Anastas (1998) highlights the Edgewise Compression Test (ECT) as critical for predicting box strength, with simplified formulas aiding industrial application. Jamsari *et al.* (2020) examine pre-crushing effects on corrugated fiberboard (CFB), emphasizing the need to model delamination for accurate predictions. Czechowski *et al.* (2021) integrate numerical and experimental methods to enhance stiffness modeling of asymmetrical five-layer boards, underscoring the importance of realistic geometry.

Flexural stiffness and its influence on compression strength receive particular attention. Hoon and Park (2004) and Cash and Frank (2020) analyze bending stiffness across various span lengths and flute types, demonstrating its critical role in packaging performance. Jamsari *et al.* (2019) investigate bending behavior across different orientations, revealing stiffness and maximum force trends crucial for structural optimization. Garbowski and Knitter-Piątkowska (2022) focus on five-layer boards, validating analytical stiffness predictions through experiments and finite element modeling.

Numerical homogenization thus enables efficient, accurate, and versatile analyses across diverse applications, from corrugated packaging to aerospace and recycled material performance.

CONCLUSIONS

Numerical homogenization methods have proven essential for understanding and optimizing the mechanical behavior of corrugated materials. This article has synthesized the key findings from the reviewed literature, emphasizing the importance of homogenization approaches and identifying future research opportunities.

The reviewed studies highlight the versatility of homogenization methods in analyzing and optimizing corrugated materials. Analytical methods, such as those proposed by Abbès and Guo (2010), provide efficient solutions for predicting torsional stiffness and other mechanical properties. Numerical approaches, particularly finite element modeling (Garbowski and Gajewski 2021), enable detailed simulations of material behavior under complex loading conditions. Experimental methods, including those by Bartolozzi *et al.* (2014), validate theoretical models and ensure their applicability in real-world scenarios.

The application of homogenization methods to corrugated materials has led to significant advancements in packaging design. Researchers including Suarez *et al.* (2021) and Di Russo *et al.* (2024) have demonstrated how homogenization can optimize the mechanical properties of corrugated cardboard for load-bearing and environmental performance. Moreover, multi-scale models have bridged the gap between microstructural details and macroscopic behavior, as shown in studies by Zhuang *et al.* (2019).

Homogenization methods play a pivotal role in addressing the inherent complexity of corrugated cardboard, which arises from its anisotropic nature and layered structure. By simplifying these materials into equivalent macroscopic models, researchers can efficiently predict critical properties such as bending stiffness, compressive strength, and shear stiffness (Buannic *et al.* 2003). This capability is particularly crucial in packaging applications, where lightweight yet robust materials are essential.

Moreover, homogenization has facilitated the development of innovative designs, such as hybrid corrugated cores and multi-functional panels. The integration of homogenization methods into optimization frameworks, as demonstrated by Shamsi Monsef *et al.* (2024), underscores their potential to enhance the sustainability and performance of corrugated materials in diverse applications.

While significant progress has been made, several challenges remain in the field of homogenization for corrugated materials. Future research should address the following areas:

- Integrating machine learning and artificial intelligence into homogenization frameworks could enhance predictive accuracy and computational efficiency (Cornaggia *et al.* 2023).
- Further exploration of how dynamic loading, humidity, and temperature impact homogenized properties is needed to improve the reliability of corrugated materials under varying conditions (Smardzewski and Jasińska 2017).
- Developing eco-friendly adhesives and recyclable materials for corrugated cores could align homogenization applications with global sustainability goals (Di Russo *et al.* 2024).
- Expanding the application of homogenization to hybrid materials and multi-functional panels, such as those used in aerospace and construction, offers promising avenues for innovation (Zhuang *et al.* 2019).

In conclusion, homogenization methods remain at the forefront of research on corrugated materials. By addressing the outlined challenges and leveraging advancements

in computational and material science, researchers can continue to unlock the full potential of these versatile materials.

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