Shear Analogy Stiffness Adjustment for CLT Plates

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In serviceability limit state, one-dimensional approaches are commonly used to estimate the bending stiffness and vertical displacement of cross-laminated timber (CLT) panels, such as the shear analogy method combined with mechanics of materials equations. Despite their simplicity, these equations disregard the orthotropic nature of wood's elastic properties and the actual dimensions of CLT panels (treated as beams), affecting displacement predictions. In this context, a parametric study was conducted in this paper using the finite element method. Then, symbolic regression was applied to propose a correction factor for adjusting the stiffness of CLT panels obtained using the shear analogy method. The symbolic regression model for the correction factor demonstrated high accuracy (R^2 = 0.9834). Adjusting the shear analogy stiffness with the proposed correction factor reduced the maximum error from 18% to 2% compared to the original method and numerical results. The model retained its accuracy in additional simulations, with percentage errors ranging from 0 to 1%.

DOI: 10.15376/biores.20.3.7027-7033

Keywords: Cross laminated timber; Numerical simulation, Symbolic regression model

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INTRODUCTION

The advent of engineered wood products such as Cross-Laminated Timber (CLT), composite panels made of wood lamellas glued in cross-laminated layers, has combined the intrinsic advantages of wood, such as sustainability and low self-weight, allowing for overcoming the limitations of its raw form. Thus, since it can be used as a slab and structural wall, CLT enables the construction of tall wooden buildings with a 40% reduction in carbon footprint compared to materials used in conventional construction (Younis and Dodoo 2022).

In structural designs involving CLT panels, serviceability limit state (SLS) checks often govern the design, particularly vibration and deflection criteria (Simović *et al.* 2023). In the SLS, one-dimensional approaches are commonly employed to estimate bending stiffness and, consequently, the vertical displacement of CLT panels. For instance, the shear analogy and gamma methods are often used in conjunction with the equations of mechanics of materials (Huang *et al.* 2023b). Despite their simplicity, these equations overlook the elastic properties of wood (considered an isotropic material) and the geometric properties of CLT panels (ignoring both curvatures of the panel and treating it as a beam), affecting displacement predictions. Three-dimensional simulations accurately represent structural behavior and prevent overestimating bending stiffness, but due to execution time, they are often avoided by designers.

Consequently, recent research has proposed alternative methods to the shear analogy method for analyzing CLT panels under out-of-plane loads. Huang *et al.* (2023a) proposed a simplified method based on composite beam theory, which yields results comparable to the shear analogy method but with lower computational effort. In another study, Huang *et al.* (2023b) developed a beam-layer model capable of predicting the longitudinal shear stress distribution in CLT panels. Rahman *et al.* (2020) evaluated the Timoshenko method for hybrid CLTs with hardwood cross layers and concluded that this method cannot accurately predict the shear response in such cases.

With this perspective, the need for a solution suitable for practical design while remaining simple for estimating the bending stiffness and displacement of CLT panels is evident. Hence, this study aimed to propose a correction factor (an equation derived through a symbolic regression model) to adjust the shear analogy method's bending stiffness estimates for CLT panels based on a parametric study using three-dimensional numerical simulations *via* the finite element method.

EXPERIMENTAL

This section presents the procedures used to derive the equation for correcting the stiffness of the shear analogy method. The selected wood species were *Pinus taeda* and *Eucalyptus spp.*, which are widely cultivated globally and used to produce CLT panels. Their properties, at 12% moisture content, are presented in Table 1.

Table 1. Elastic Properties of Species

Specie	Density (kg/m³)	E_L (MPa)	E_T (MPa)	E_R (MPa)	G_{LR} (MPa)	G_{LT} (MPa)	G_{RT} (MPa)	ν_{LR}	ν_{LT}	ν_{RT}
Pinus taeda[1]	510	12300.0	959.40	1389.90	1008.6	996.30	159.90	0.33	0.29	0.38
Eucalyptus spp. [2]	924.8	21767.5	827.17	2133.22	1458.4	2046.15	544.19	0.42	0.60	0.38

[1] values provided by Kretschmann (2010); [2] values provided by Crespo *et al.* (2017) and Martins (2018)

Shear Analogy

The shear analogy method is a technique used to calculate the effective stiffness of structural elements made of wood with composite sections, such as CLT panels. In this methodology, the effective stiffness of the panel (Eq. 1) is obtained by summing the individual stiffness of each layer along with the term known as the Steiner term ($E_i \cdot A_i \cdot a_i^2$),

$$EI_{ef,S} = \sum_{i=1}^{n} E_i I_i + E_i A_i a_i^2 \quad (1)$$

where $EI_{ef,S}$ (N·mm^2) is the bending stiffness obtained by the shear analogy method, n is the number of layers, i is the index referring to the layers, E_i (N/mm²) is the modulus of elasticity of the i-th layer, I_i (mm^4) is the moment of inertia of the i-th layer, A_i (mm^2) is the cross-sectional area of the i-th layer, and a_i (mm) is the distance from the center of gravity of the element to the center of gravity of the i-th layer.

Using the effective stiffness from the shear analogy method, the maximum vertical displacement at the center of the span of a simply supported CLT panel under a uniformly distributed load can be estimated through Eq. 2. Additionally, when estimating the loading (Eq. 3), a limiting displacement (w_{lim}) can be adopted as $L/500$, as given in the Eurocode 5 standard (EN 1995-1-1 2004).

$$w = \frac{5 \cdot q \cdot L^4}{384 \cdot EI} \tag{2}$$

$$q_{lim} = \frac{w_{lim} \cdot 384 \cdot EI}{5 \cdot L^4} \tag{3}$$

In Eqs. 2 and 3, w (mm) is the maximum vertical displacement at the center of the span, q (N/mm) is the uniformly distributed load per unit length, L (mm) is the length of the element, and EI (N·mm^2) is the bending stiffness obtained by the shear analogy method. It is worth noting that this equation only considers bending for displacement calculation, neglecting the shear component.

Numerical Simulation

In the parametric study of CLT panels, numerical simulations were conducted to analyze the influence of the length (3500, 5000, and 16500), layer configurations (Table 2), and the elastic properties of wood species (Table 1) on the stiffness values obtained by the shear analogy method ($EI_{ef,S}$) and the numerical simulations (EI_{num}) performed in the Abaqus software. Thus, for each of the 54 compositions, it was possible to obtain the correction coefficient (β) through the difference between the stiffness values ($EI_{ef,S}$ and EI_{num}), as shown in Fig. 1.

Table 2. Layer Configurations

Configuration	Number of layers	CLT plate thickness (mm)	Thickness (mm) and Layer Orientation						
			L	T	L	T	L	T	L
3L [60]	3	60	20	20	20	-	-	-	-
3L [90]	3	90	30	30	30	-	-	-	-
3L [120]	3	120	40	40	40	-	-	-	-
5L [140]	5	140	40	20	20	20	40	-	-
5L [160]	5	160	40	20	40	20	40	-	-
5L [180]	5	180	40	30	40	30	40	-	-
7L [200]	7	200	20	40	20	40	20	40	20
7L [220]	7	220	30	40	30	20	30	40	30
7L [240]	7	240	30	40	30	40	30	40	30

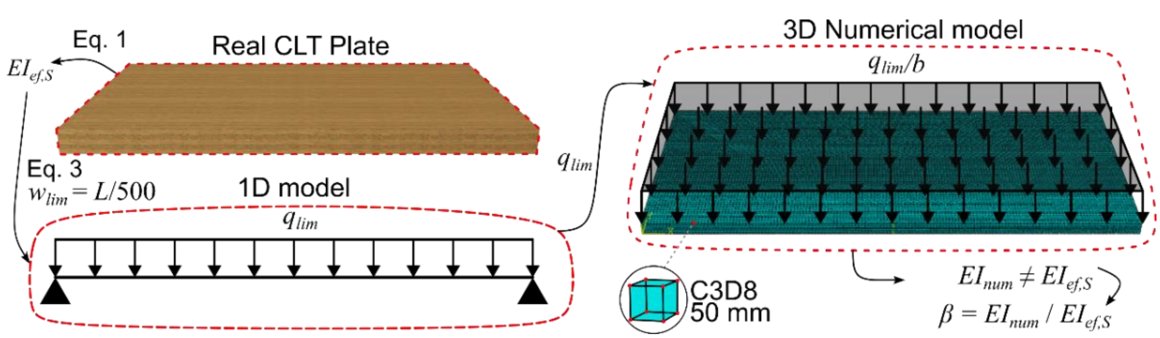

Fig. 1. Overview of the numerical model for the CLT plate

Thus, for each combination, the effective stiffness was calculated using the shear analogy method (Eq. 2). Consequently, using Eq. 3, the uniform loading that would result in the limiting displacement (L/500) was calculated (Fig. 1). Considering the SLS, the CLT panels were modeled with continuous layers that were perfectly bonded. The chosen finite element was the C3D8 due to the rectangular shape of the CLT panels, with an average edge size of 50 mm (Fig. 1). Additionally, a simply supported configuration was adopted for the panels, with the loads being previously calculated. From the numerical simulations, the vertical displacement at the center of the span (w_{num}), along with the other variables, was used to calculate the stiffness of the numerical models of the CLT panels (Eq. 4).

$$EI_{num} = \frac{5 \cdot q_{lim} \cdot L^4}{384 \cdot w_{num}} \quad (4)$$

Accordingly, through the ratio of the numerical stiffness (EI_{num}) to the stiffness from the shear analogy method ($EI_{ef,S}$), it was possible to obtain the correction factor values (β) for each CLT panel composition (Fig. 1). Thus, the adjusted stiffness (EI_{adj}) can be calculated according to Eq. 5.

$$EI_{adj} = \beta \cdot EI_{ef,S} \quad (5)$$

Symbolic Regression Model

Symbolic regression was used to obtain an accurate regression model for estimating the correction factor β. Based on Darwin's theory of evolution, evolutionary concepts are used to find precise regression equations in symbolic regression. Through the definition of variables and mathematical operators, the algorithm starts with random equations, selects the most accurate ones, and generates new generations from them. This cycle continues until the most suitable equation for the solution is identified (Cranmer 2023).

In this study, the PySR Python library was used for symbolic regression. The variables considered were the plate length (L), the sum of the thicknesses of the transverse layers (h_T), the sum of the thicknesses of the longitudinal layers (h_L), the total plate thickness (h), the number of layers (n_c), and the wood's elastic properties (E_L, E_T, E_R, G_{LR}, G_{LT}, G_{RT}, ν_{LR}, ν_{LT}, ν_{RT}). The mathematical operators used were addition, multiplication, subtraction, and division, while the unary operators were quadratic and cubic. To obtain a simple and accurate equation, the parameter "max-Size" was set to 25, and "niterations" was set to 100,000. Furthermore, additional simulations with new lengths (5000 and 12000 mm) and section configurations used in constructing the equation (Table 2) were considered to evaluate the accuracy of the proposed equation when extrapolated.

RESULTS AND DISCUSSION

In Fig. 2, the correction factor (β) values can be observed for each combination of the variables considered in the parametric study (length and layer configuration). Among the values for β, 81.48% of the results showed ratios below 1, while 18.52% were above 1. Consequently, in most cases (81.48%), the stiffness obtained using the shear analogy method was higher than that calculated by the numerical model. As a result, the displacement determined by the traditional method was smaller than the numerical one.

The correction factor (β) showed minor variations with changes in configurations for lengths of 8000 mm and 16500 mm. However, for the shortest length (3500 mm), a significant deviation of β from 1 was observed, as the layer configurations varied with increasing thickness (Fig. 2). This outcome can be explained by the exclusion of shear effects in the equation of solid mechanics (Eq. 2), which led to smaller displacements compared to those obtained from the numerical model, particularly in scenarios where shear effects are significant, such as for shorter plate lengths (3500 mm).

When analyzing the influence of species on β, despite the similar behavior of the curves (Fig. 2), the *Pinus taeda* species required greater corrections than *Eucalyptus spp*. This outcome can be explained by the fact that *Eucalyptus spp*. has higher elastic properties, which resulted in higher numerical stiffness values (EI_{num}) and, consequently, a smaller difference between the numerical stiffness (EI_{num}) and the stiffness from the shear analogy method ($EI_{ef,S}$).

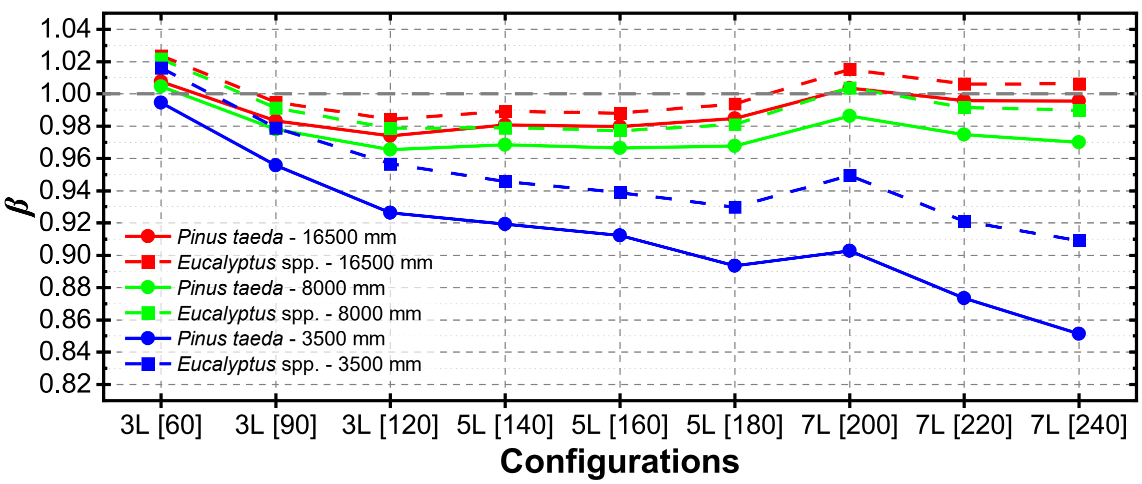

Fig. 2. Correction factor values

The regression model of the correction coefficient β for both species, obtained through Symbolic Regression, can be seen in Eq. 6,

$$\beta = \left(-\frac{v_{LR}\left(n_c + \left(\frac{b}{L}\right)^3\right)}{h_L + n_c} + \left(\frac{\frac{h}{v_{LR}} + 237.913}{L}\right)^2 - 0.979 \right)^2 \quad (1)$$

where β is the correction factor, μ_{LR} is the Poisson's ratio in the longitudinal-radial plane, n_c is the number of layers, b (mm) is the plate width, L (mm) is the plate length, h_L (mm) is the sum of the longitudinal layer thicknesses, and h (mm) is the total plate thickness. It is worth noting that to calculate β (Eq. 6), b is set to 3500, and v_{LR} must be 0.33 for *Pinus taeda* and 0.42 for *Eucalyptus* spp, respectively. The other geometric variables may vary within the ranges indicated in Eq. 7.

$$\beta \text{ with } n_c \in [1,7],\ L \in [3500,\ 16500],\ h_L \in [40,\ 120], \text{ and } h \in [60,\ 240] \quad (7)$$

The symbolic regression model shown in Eq. 6 exhibited high accuracy in estimating β, with a coefficient of determination (R^2) of 0.9834, mean absolute percentage error (MAPE) of 0.425%, and mean squared error (MSE) of $2.32 \cdot 10^{-5}$. This excellent precision in estimating β enabled the shear analogy method stiffness values to be aligned with the numerical results. Therefore, it is worth noting that the adjusted stiffness values can be obtained using Eq. 5.

The histogram in Fig. 3 illustrates the adjustment quality, which shows the density of percentage errors for both the original stiffness values and the adjusted (obtained using Eq. 5) ones compared to the numerical results. While the errors for the adjusted values are between 0 and 2%, the original shear analogy method errors range from 0 to 18%. For the errors in the adjusted values using β, 90.74% of the data resulted in errors below 0.75%, 98.15% in errors up to 1%, and 100% in errors up to 1.1%.

When additional simulations were considered, the proposed equation showed good precision in adjusting the shear analogy method stiffness values, with a MAPE of 0.435% compared to the numerical results. The accuracy of the equation when extrapolated is highlighted in Fig. 3b, which shows the concentration of percentage errors for the proposed equation between 0 and 2%. In comparison, the traditional method exhibits errors of up to 9%.

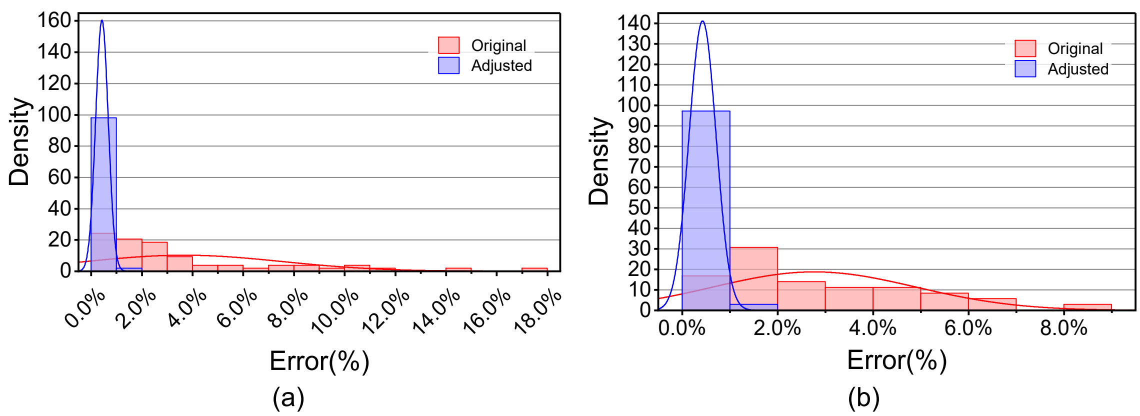

Fig. 3. Histograms of percentage errors for values (a) within the model's range and (b) outside the model's range

CONCLUSIONS

1. The correction factor β showed values below 1 in 81.48% of the results, indicating that, in most cases, the displacement estimated by the traditional method was lower than the numerical results. This outcome can be attributed to the neglect of shear effects in the mechanics of materials equation, leading to lower displacements than the numerical model, especially for shorter plates (3500 mm).
2. The regression model demonstrated high accuracy in estimating the correction factor (β), with an R^2 of 0.9834, a MAPE of 0.425%, and an MSE of $2.32 \cdot 10^{-5}$. Additionally, 90.74% of the data used in the model's construction resulted in errors below 0.75%. When comparing the original method with the adjusted one based on numerical results, the correction factor reduced the maximum error from 18% to 2%. Furthermore, even considering additional simulations, the regression model accurately estimated β, with percentage errors ranging from 0 to 1%.

ACKNOWLEDGMENTS

The authors are grateful for the support of the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior, Brazil (CAPES).

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Article submitted: December 16, 2024; Peer review completed: March 8, 2025; Revised version received: March 17, 2025; Accepted: April 15, 2025; Published: July 3, 2025.
DOI: 10.15376/biores.20.3.7027-7033