Seismic Performance Analysis of Wood-Steel Frame Structures

Panxu Sun, Keguang Li, and Gangzhu Sun *

Wood and steel have different physical properties. By combining the two kinds of materials as bonded parallel beams, wood-steel frame structures can be prepared. This work considers the seismic performance of such engineering structures. Based on the Rayleigh damping model of substructures, the non-proportional damping coefficient is used to quantify the structural non-proportional damping characteristic of wood-steel frame structures. A complex mode superposition method is used to calculate seismic responses. Numerical cases showed that compared with the frame structure with upper steel and lower wood, the overall lateral performance is lower and the local lateral performance is higher for the frame structure with upper wood and lower steel. The overall lateral seismic design needs to be improved for the wood-steel frame structure with upper wood and lower steel. The local lateral seismic design needs be improved for the wood-steel frame structure with upper steel and lower wood.

DOI: 10.15376/biores.19.4.8730-8738

Keywords: Wool-steel; Frame structure; Non-proportional damping; Seismic analysis; Lateral performance

*Contact information: School of Civil Engineering, Zhengzhou University, Zhengzhou, 450001, China; *Corresponding author: gzsun@zzu.edu.cn*

INTRODUCTION

Wood has advantages of light weight and high strength. These attributes help make it an important building material for sustainable development (Pramreiter *et al.* 2023). The wood structures have good tensile-compressive and bending performance (Sun *et al.* 2023). Steel has the advantages of high rigidity and high ductility (Işık and Özdemir 2022). Combining the advantages of the two kinds of materials, wood-steel composite members are widely constructed, such as wood-steel barriers (Goubel *et al.* 2012), as well as wood-steel joists (Wu *et al.* 2022). Kaliyanda *et al.* (2019) compared numerical results with test information and constructed a three-dimensional finite-element model of wood-steel bolted joints. Conrad and Phillips (2019) analyzed the shear behavior wood-steel composite shear wall. The composite shear wall was shown to develop a large amount of shear capacity over a short wall length. Wu *et al.* (2021) combined with experimental results, studied the capacity of I-shaped wood-steel beam. Li *et al.* (2014) analyzed seismic performance of the timber-steel hybrid shear wall systems with an infill wood shear wall.

Recent studies of the wood-steel structures mainly have aimed at component level and structural level. Firstly, the research of the wood-steel component level is developed. Some wood-steel components are further designed, such as wood-steel beams, wood-steel columns, wood-steel walls, *etc*. Secondly, the research of the wood-steel structural level is developed. Some composite structures that consist of wood substructure and steel substructure are designed in the engineering structures, such as wood-steel frame structures (López Almansa *et al.* 2012) and wood-steel bridges (Gocál *et al.* 2024). Wood and steel are further applied in frame structures, for which the different material damping characteristics of the two materials can be expected to affect the overall performance. Wood-steel frame structures can be regarded as composite structures, which have non-proportional damping characteristics. There have been relatively few studies on the dynamic characteristics of wood-steel structures. Therefore, seismic performance analysis of wood-steel frame structures is important for seismic design of engineering structures.

This study analyzed seismic performance of wood-steel frame structures. First, the Rayleigh damping model of substructures was used to construct the damping matrix of wood-steel frame structures. The non-proportional damping coefficient was used to quantify the structural non-proportional damping characteristic, and a complex mode superposition method was used to calculate seismic responses. Finally, the different vertical distribution of wood and steel for frame structures were taken as numerical examples. The overall and local lateral stiffness were further analyzed.

EXPERIMENTAL

Time-Domain Dynamic Equation of Wood-Steel Frame Structures

The time-domain dynamic equation of wood-steel frame structures can be expressed as follows,

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t)+\boldsymbol{C}\dot{\boldsymbol{x}}(t)+\boldsymbol{K}\boldsymbol{x}(t)=\boldsymbol{F} \quad (1)$$

where $\boldsymbol{M}$ is the mass matrix, $\boldsymbol{C}$ is the damping matrix, and $\boldsymbol{K}$ is the stiffness matrix.

The mass matrix of wood-steel frame structures is expressed as follows,

$$\boldsymbol{M}=\begin{bmatrix}\boldsymbol{M}_{\mathrm{W}} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{M}_{\mathrm{S}}\end{bmatrix} \quad (2)$$

where $\boldsymbol{M}_{\mathrm{W}}$ is the mass matrix of wood substructure, and $\boldsymbol{M}_{\mathrm{S}}$ is the mass matrix of steel substructure.

The stiffness matrix of wood-steel frame structures is expressed as follows,

$$\boldsymbol{K}=\begin{bmatrix}\boldsymbol{K}_{\mathrm{S}} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{K}_{\mathrm{W}}\end{bmatrix} \quad (3)$$

where $\boldsymbol{K}_{\mathrm{W}}$ is the stiffness matrix of wood substructure, and $\boldsymbol{K}_{\mathrm{S}}$ is the stiffness matrix of steel substructure.

The damping matrix of wood-steel frame structures is expressed as follows,

$$\boldsymbol{C}=\begin{bmatrix}\boldsymbol{C}_{\mathrm{S}} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{C}_{\mathrm{W}}\end{bmatrix} \quad (4)$$

where $\boldsymbol{C}_{\mathrm{W}}$ is the damping matrix of wood substructure, and $\boldsymbol{C}_{\mathrm{S}}$ is the damping matrix of steel substructure.

Based on the Rayleigh damping model (Spears and Jensen 2012), the damping matrix can be expressed in Eqs. 5 through 8,

$$\boldsymbol{C}_{\mathrm{W}} = \alpha_{\mathrm{W}} \boldsymbol{M}_{\mathrm{W}} + \beta_{\mathrm{W}} \boldsymbol{K}_{\mathrm{W}} \tag{5}$$

$$\boldsymbol{C}_{\mathrm{S}} = \alpha_{\mathrm{S}} \boldsymbol{M}_{\mathrm{S}} + \beta_{\mathrm{S}} \boldsymbol{K}_{\mathrm{S}} \tag{6}$$

where α_{W} and β_{W} are Rayleigh damping coefficients of wood substructure, and α_{S} and β_{S} are Rayleigh damping coefficients of steel substructure;

$$\begin{cases} \alpha_{\mathrm{W}} = \dfrac{\omega_m \omega_n (2\xi_{\mathrm{W}}\omega_m - 2\xi_{\mathrm{W}}\omega_n)}{\omega_m^2 - \omega_n^2} \\ \beta_{\mathrm{W}} = \dfrac{2\xi_{\mathrm{W}}\omega_m - 2\xi_{\mathrm{W}}\omega_n}{\omega_m^2 - \omega_n^2} \end{cases} \tag{7}$$

$$\begin{cases} \alpha_{\mathrm{S}} = \dfrac{\omega_m \omega_n (2\xi_{\mathrm{S}}\omega_m - 2\xi_{\mathrm{S}}\omega_n)}{\omega_m^2 - \omega_n^2} \\ \beta_{\mathrm{S}} = \dfrac{2\xi_{\mathrm{S}}\omega_m - 2\xi_{\mathrm{S}}\omega_n}{\omega_m^2 - \omega_n^2} \end{cases} \tag{8}$$

where ω_m is natural frequency of the $m^{\text{-th}}$ mode, ω_n is natural frequency of the $n^{\text{-th}}$ mode, ξ_{W} is damping ratio of wood, and ξ_{S} is damping ratio of steel.

Non-Proportional Damping Characteristics of Wood-Steel Frame Structures

Based on M and K, the undamped modal vectors of wood-steel frame structures can be expressed in Eq. 9.

$$\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{\phi}_1 & \boldsymbol{\phi}_2 & \cdots & \boldsymbol{\phi}_N \end{bmatrix} \tag{9}$$

Based the orthogonality of vibration mode vectors, the modal mass matrix and modal stiffness matrix of wood-steel frame structures can be expressed as diagonal matrices, namely:

$$\boldsymbol{\Phi}^{\mathrm{T}} \boldsymbol{M} \boldsymbol{\Phi} = \begin{bmatrix} m_{11} & & & \\ & m_{22} & & \\ & & \ddots & \\ & & & m_{NN} \end{bmatrix} \tag{10}$$

$$\boldsymbol{\Phi}^{\mathrm{T}} \boldsymbol{K} \boldsymbol{\Phi} = \begin{bmatrix} k_{11} & & & \\ & k_{22} & & \\ & & \ddots & \\ & & & k_{NN} \end{bmatrix} \tag{11}$$

The damping ratio of wood is different from that of steel, and the wood-steel frame structure is non-proportionally damped system. The modal stiffness matrix can be expressed as the diagonal matrix, as follows.

$$\boldsymbol{\Delta} = \boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{C}\boldsymbol{\Phi} = \begin{bmatrix} \Delta_{11} & \Delta_{12} & \cdots & \Delta_{1N} \\ \Delta_{21} & \Delta_{22} & \cdots & \Delta_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \Delta_{N1} & \Delta_{N2} & \cdots & \Delta_{NN} \end{bmatrix} \tag{12}$$

The diagonal matrix of Δ is shown in Eq. 13.

$$\boldsymbol{D} = \begin{bmatrix} \Delta_{11} & & & \\ & \Delta_{22} & & \\ & & \ddots & \\ & & & \Delta_{NN} \end{bmatrix} \tag{13}$$

Based on $D^{-1}\Delta$, the non-proportional damping coefficient (Tong *et al.* 1994) is expressed as follows,

$$\delta = \frac{\lambda_{\max} - \lambda_{\min}}{\lambda_{\max} + \lambda_{\min}} \tag{14}$$

where λ_{max} is the maximum eigenvalue of $D^{-1}\Delta$, and λ_{min} is the corresponding minimum eigenvalue of $D^{-1}\Delta$.

The non-proportional damping coefficient ranges from 0 to 1. A larger coefficient indicates a greater non-proportional characteristic. When the non-proportional damping coefficient is 0, the frame structure will be the proportionally damped system.

Numerical Method of Seismic Responses for Wood-Steel Frame Structures

For an earthquake wave, the time-domain dynamic equation of wood-steel frame structures is expressed as follows,

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t) + \boldsymbol{C}\dot{\boldsymbol{x}}(t) + \boldsymbol{K}\boldsymbol{x}(t) = -\boldsymbol{M}\boldsymbol{I}g(t) \tag{15}$$

where I is the distribution vector, $g(t)$ is the acceleration of earthquake wave.

The state space method is adopted (Foss 1958). The auxiliary equation is introduced in Eq. 16.

$$\boldsymbol{M}\dot{\boldsymbol{x}}(t) - \boldsymbol{M}\dot{\boldsymbol{x}}(t) = \boldsymbol{0} \tag{16}$$

Equations 15 and 16 can be combined as Eq. 17,

$$\boldsymbol{P}\dot{\boldsymbol{y}}(t) + \boldsymbol{Q}\boldsymbol{y}(t) = \boldsymbol{R} \tag{17}$$

where

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{C} & \boldsymbol{M} \\ \boldsymbol{M} & \boldsymbol{0} \end{bmatrix} \tag{18}$$

$$\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{K} & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{M} \end{bmatrix} \tag{19}$$

$$\boldsymbol{y}(t) = \begin{bmatrix} \boldsymbol{x}(t) \\ \dot{\boldsymbol{x}}(t) \end{bmatrix} \tag{20}$$

$$\boldsymbol{R} = \begin{bmatrix} -\boldsymbol{MI}g(t) \\ \boldsymbol{0} \end{bmatrix} \tag{21}$$

The complex eigenvectors of Eq. (17) are

$$\boldsymbol{\Psi} = \begin{bmatrix} \boldsymbol{\psi}_i & \boldsymbol{\psi}_i^* \\ \lambda_i \boldsymbol{\psi}_i & \lambda_i^* \boldsymbol{\psi}_i^* \end{bmatrix} \quad (i = 1, 2, \ldots, N) \tag{22}$$

where $\boldsymbol{\psi}_i$ is N-dimensional complex vector, $\boldsymbol{\psi}_i$* is conjugate vector of $\boldsymbol{\psi}_i$, λ_i is eigenvalue, and λ_i* is conjugate value of λ_i.

Equation 22 is substituted into Eq. 17, which is obtained as Eq. 23,

$$\dot{z}_i(t) - \lambda_i z_i(t) = -\gamma_i g(t) \quad (i = 1, 2, \ldots, N) \tag{23}$$

where

$$\gamma_i = \frac{\boldsymbol{\psi}_i^{\mathrm{T}} \boldsymbol{MI}}{\boldsymbol{\psi}_i^{\mathrm{T}} \boldsymbol{C} \boldsymbol{\psi}_i + 2\lambda_i \boldsymbol{\psi}_i^{\mathrm{T}} \boldsymbol{M} \boldsymbol{\psi}_i} \tag{24}$$

The displacement vectors can be represented linearly by complex eigenvectors, *i.e.*,

$$\boldsymbol{x}(t) = \sum_{i=1}^{N} \left[\boldsymbol{\psi}_i z_i(t) + \boldsymbol{\psi}_i^* z_i^*(t) \right] \tag{25}$$

Solving Eq. 23, $z_i(t)$ can be obtained, and $x(t)$ is further obtained from Eq. 25. The complex mode superposition method is realized, which can be regarded as the numerical method of seismic responses for wood-steel frame structures.

RESULTS AND DISCUSSION

Vertical hybrid structures can be equivalent to a two degree of freedom system (Chen and Wu 1999; Huang *et al.* 2015). The two degree of freedom system for a wood-steel frame structure is taken as a numerical example. Model A is the frame structure with upper wood and lower steel and Model B is the frame structure with upper steel and lower wood. The mass distribution, stiffness distribution and damping distribution are shown in Fig. 1. Here the stiffness distributions of Model A and Model B are the same. Besides, the ratios of mass to stiffness for steel and wood of Model A are same as those of Model B. The mass, stiffness and damping ratio are shown in Table 1.

In numerical models, the ratio of mass to stiffness for wood is 1/3, and the ratio of mass to stiffness for steel is 2/3. According to Standard for Design of Timber Structures GB/T 50005 (2017), the damping ratio of wood is 0.05. According to Code for Seismic Design of Buildings GB 50011 (2010), the damping ratio of steel is 0.02. Adopting the quantification method, the non-proportional damping coefficients of Model A and Model B are 0.3800 and 0.4147, respectively.

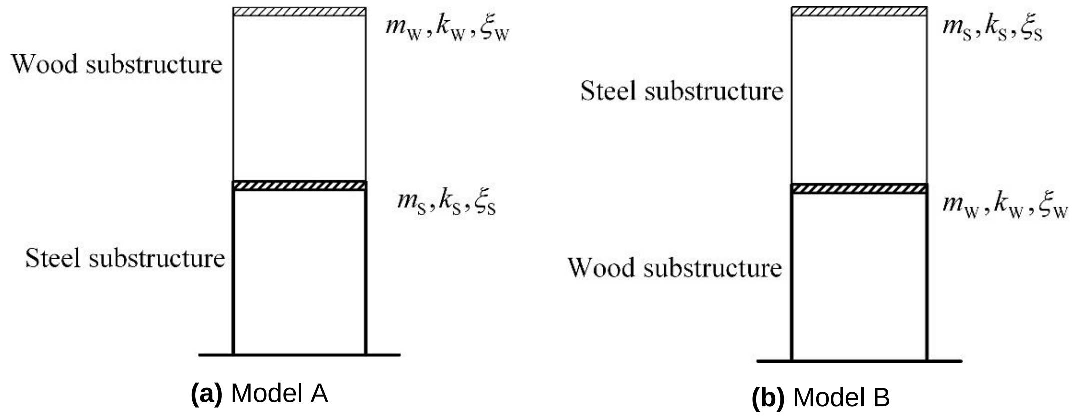


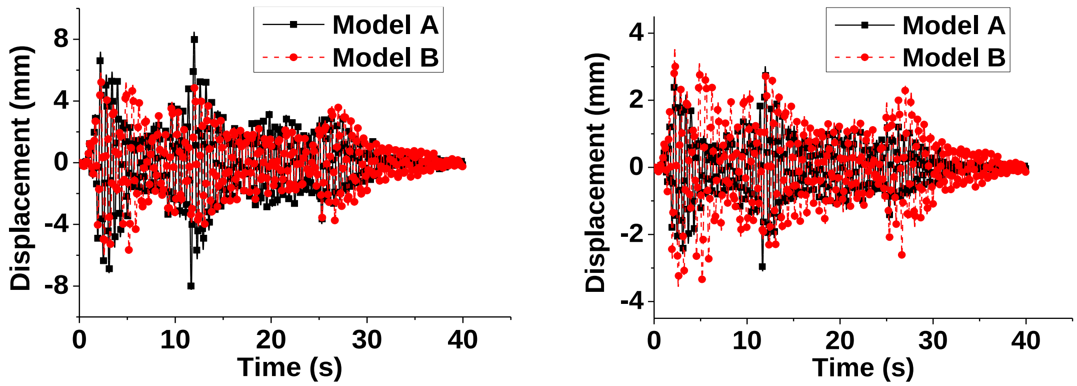
Fig. 1. Diagrams of wool-steel frame structures

Table 1. Parameters of Wool-Steel Frame Structures for Model A and Model B

	m_W (kg)	k_W (N/m)	ξ_W	m_S(kg)	k_S (N/m)	ξ_S
Model A	1000	3.0×10^5	0.05	4000	6.0×10^5	0.02
Model B	2000	6.0×10^5	0.05	2000	3.0×10^5	0.02

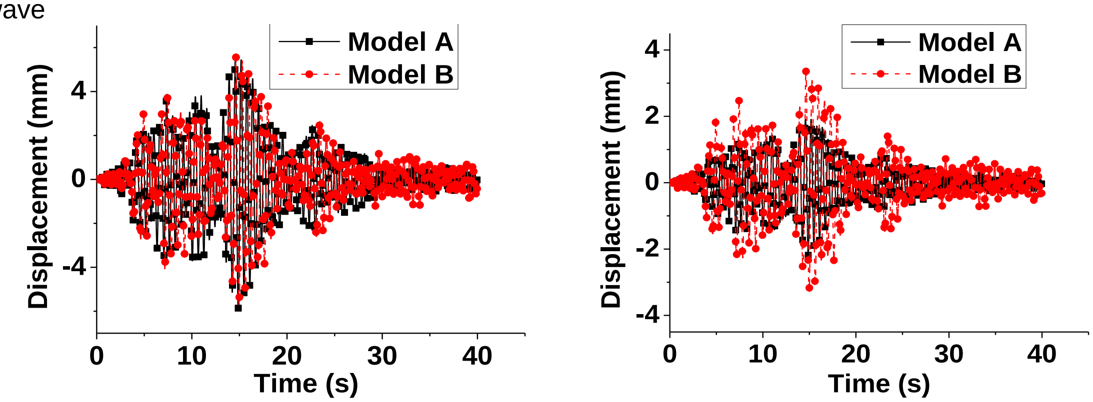
The El Centro wave and Taft wave were used to calculate the seismic responses for wood-steel frame structures. The proposed numerical method was adopted. The time-domain displacement responses of Model A and Model B are shown in Figs. 2 and 3. The comparative analysis of displacement responses is shown in Table 2. In the seismic design of frame structures, the peak top displacement and peak base shear force can be defined as the indexes of overall lateral performance, and the peak story drift can be defined as the indexes of local lateral performance. The vertical distribution of wood and steel of Model A is different from that of Model B. The seismic performance with different vertical distribution of wood and steel can be quantitatively analyzed.

The peak top displacement of Model A was larger than that of Model B. The relative errors for El Centro wave and Taft wave were 29.65% and 3.17%, respectively. The peak base shear force of Model A was larger than that of Model B. The relative errors for El Centro wave and Taft wave were 39.41% and 15.28%, respectively. Therefore, the overall lateral performance of Model A was lower than that of Model B. Compared with the frame structure with upper steel and lower wood, the overall lateral performance was lower for the frame structure with upper wood and lower steel. However, the peak story drift of Model A was less than that of Model B. The relative errors for El Centro wave and Taft wave were 14.88% and 52.87%, respectively. Compared with Model B, the local lateral performance was higher for Model A. Compared with the frame structure with upper steel and lower wood, the local lateral performance was higher for the frame structure with upper wood and lower steel. The reason is that the elastic modulus and density of wood are less than those of steel, and the damping ratio of wood is larger than that of steel. Therefore, the vertical distribution of wood and steel is important for wood-steel frame structures. The overall lateral seismic design needs to be improved for the wood-steel frame structure with upper wood and lower steel. The local lateral seismic design needs to be improved for the wood-steel frame structure with upper steel and lower wood.



(a) Top displacement **(b)** Story drift

Fig. 2. Time-domain displacement responses of wood-steel frame structures due to El Centro wave



(a) Top displacement **(b)** Story drift

Fig. 3. Time-domain displacement responses of wood-steel frame structures due to Taft wave

Table 2. Comparative Analysis of Displacement Responses for Models A and B

	El Centro Wave		Taft Wave	
	Model A	Model B	Model A	Model B
Peak top displacement (mm)	8.4891	5.9724	5.8917	5.7047
Relative error (%)	—	29.65	—	3.17
Peak story drift (mm)	3.0985	3.5595	2.1949	3.3553
Relative error (%)	—	14.88	—	52.87
Peak base shear force (kN)	4.1894	2.5384	2.8765	2.4371
Relative error (%)	—	39.41	—	15.28

CONCLUSIONS

In this paper, the calculation method of seismic responses for wood-steel structures was constructed. Then the seismic performance of wood-steel frame structures with different vertical distribution of wood and steel were analyzed.

1. The A time-domain numerical method of wood-steel frame structures was constructed based on Rayleigh damping model of substructures. It was found that the non-proportional damping coefficient can quantitatively analyze structural non-proportional damping characteristic, and the corresponding complex mode superposition method can be used to calculate seismic responses.

2. The overall and local lateral performances are important in seismic design of wood-steel frame structure. The vertical distribution of wood and steel affects the lateral performance of wood-steel frame structures. Compared with the frame structure with upper steel and lower wood, the overall lateral performance was lower and the local lateral performance was higher for the frame structure with upper wood and lower steel. The overall lateral seismic design needs to be improved for the wood-steel frame structure with upper wood and lower steel. The local lateral seismic design needs to be improved for the wood-steel frame structure with upper steel and lower wood.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (Grant No. 52208322) and the Key Research Project of Henan Higher Education Institutions (Grant No. 22A560005).

REFERENCES CITED

Chen, G., and Wu, J. (1999). "Transfer-function-based criteria for decoupling of secondary systems," *Journal of Engineering Mechanics* 125(3), 340-346. DOI: 10.1061/(ASCE)0733-9399(1999)125:3(340)

Conrad, K., and Phillips, A. R. (2019). "Full scale testing and development of wood-steel composite shear walls," *Structures* 20, 268-278. DOI: 10.1016/j.istruc.2019.04.010

Foss, K. A. (1958). "Coordinates which uncouple the equations of motion of damped linear dynamic systems," *Journal of Applied Mechanics* 25, 361-364.

GB 50011 (2010). "Code for seismic design of buildings," Standardization Administration of China, Beijing, China.

GB/T 50005 (2017). "Standard for design of timber structures," Standardization Administration of China, Beijing, China.

Gocál, J., Vičan, J., Odrobiňák, J., Hlinka, R., Bahleda, F., and Wdowiak-Postulak, A. (2024). "Experimental and numerical analyses of timber-steel footbridges," *Applied Sciences* 14, article 3070. DOI: 10.3390/app14073070

Goubel, C., Massenzio, M., and Ronel, S. (2012). "Wood-steel structure for roadside safety barriers," *International Journal of Crashworthiness* 17(1), 63-73. DOI: 10.1080/13588265.2011.625678

Huang, W., Qian, J., Zhou, Z., and Fu, Q. (2015). "An approach to equivalent damping ratio of vertically mixed structures based on response error minimization," *Soil Dynamics and Earthquake Engineering* 72, 119-128. DOI: 10.1016/j.soildyn.2015.02.008

Işık, E., and Özdemir, M. (2017). "Performance based assessment of steel frame structures by different material models," *International Journal of Steel Structures* 17, 1021-1031. DOI: 10.1007/s13296-017-9013-x

Kaliyanda, A. R., Rammer, D. R., and Rowlands, R. E. (2019). "Three-dimensional nonlinear finite-element analysis of wood–steel bolted joints subjected to large deformations," *Journal of Structural Engineering* 145(10), article 04019108. DOI: 10.1061/(ASCE)ST.1943-541X.0002036

Li, Z., He, M., Li, M., and Lam, F. (2014). "Damage assessment and performance-based seismic design of timber-steel hybrid shear wall systems," *Earthquakes and Structures* 7, 101-117. DOI: 10.12989/eas.2014.7.1.101

López Almansa, F., Segués, E., and Cantalapiedra, I. R. (2015). "A new steel framing system for seismic protection of timber platform frame buildings. Implementation with hysteretic energy dissipators," *Earthquake Engineering and Structural Dynamics* 44, 1181-1202. DOI: 10.1002/eqe.2507

Pramreiter, M., Nenning, T., Malzl, L., and Konnerth, J. (2023). "A plea for the efficient use of wood in construction," *Nature Reviews Materials* 8, 217-218. DOI: 10.1038/s41578-023-00534-4.

Spears, R. E., and Jensen, S. R. (2012). "Approach for selection of Rayleigh damping parameters used for time history analysis," *Journal of Pressure Vessel Technology* 134(6), article 061801. DOI: 10.1115/1.4006855

Sun, P., Liang, K., and Wang, S. (2023). "Comparative analysis of mechanical properties for wood frame and reinforced concrete frame based on deformation energy decomposition method," *BioResources* 18(4), 7124-7142. DOI: 10.15376/biores.18.4.7124-7142

Tong, M., Liang, Z., and Lee, G. C. (1994). "An index of damping non-proportionality for discrete vibrating systems," *Journal of Sound Vibration* 174(1), 37-55. DOI: 10.1006/jsvi.1994.1554

Wu, G., Zhu, E., Ren, H., Zhong, Y., and Gong, M. (2022). "Application of small-diameter round timber as structural members in light frame construction," *Journal of Asian Architecture and Building Engineering* 21, 1029-1039. DOI: 10.1080/13467581.2021.1929241

Wu, S., Shan, Q., Zhang, J., Tong, K., and Li, Y. (2021). "Shear behavior of I-shaped wood-steel composite beam," *BioResources* 16(1), 583-596. DOI: 10.15376/biores.16.1.583-596

Article submitted: August 15, 2024; Peer review completed: September 7, 2024; Revised version received: September 12, 2024; Accepted: September 13, 2024; Published: October 1, 2024.
DOI: 10.15376/biores.19.4.8730-8738